# MATHEMATICS Preparation 10

## Exploration of Numbers

## Module 1

Learning Technologies Branch

Alberta LEARNING

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Mathematics Preparation 10
Module 1: Exploration of Numbers
Student Module Booklet
Learning Technologies Branch
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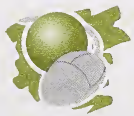
| This document is intended for | |
|---|---|
| Students | ✓ |
| Teachers | ✓ |
| Administrators | |
| Parents | |
| General Public | |
| Other | |



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**http://www.learning.gov.ab.ca/ltb**

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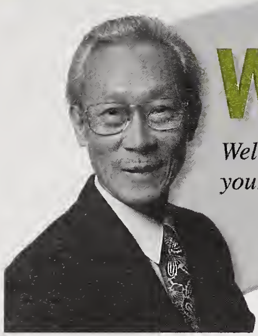


# Welcome

*Welcome to Module 1. We hope you enjoy your study of "Exploration of Numbers."*

## Course Overview

Mathematics Preparation 10 consists of five modules and a final test. Each module is worth 1 credit. Discuss with your teacher how many modules you need to complete to prepare yourself for Pure Mathematics 10 or Applied Mathematics 10.

**Mathematics Preparation 10**

**Module 1**
Exploration of Numbers

**Module 2**
Number Connections

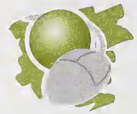
**Module 3**
Patterns and Algebra

**Module 4**
Polynomials

**Module 5**
Shape and Space

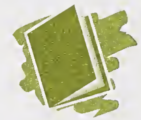
The document you are now reading is called a Student Module Booklet. It has accompanying Assignment Booklets.

You will find many visual cues or icons throughout this Student Module Booklet. Read the following statements to discover what each icon prompts you to do.


Use the Internet to explore a topic.


Use a scientific calculator.


Use the suggested answers in the Appendix to correct activities.


Answer the questions in the Assignment Booklet.

Throughout the course, you will be given instruction and practice on each of the following mathematical processes:

- connecting mathematical ideas
- developing and using estimation strategies
- developing and using mental math strategies
- using technology appropriately
- developing and using problem-solving strategies
- reasoning and justifying your answers
- using visualization to assist in processing information
- solving problems
- communicating mathematically

For example, to help you develop your problem-solving skills, several problem-solving strategies are explained in the Appendix of each Student Module Booklet, and you will be given several non-routine problems to solve in each module. Watch for the heading "Now Try This."

To help you develop your communication skills in mathematics, you will be asked to do several journal writings in each module. Watch for the heading "Looking Back."

# Contents



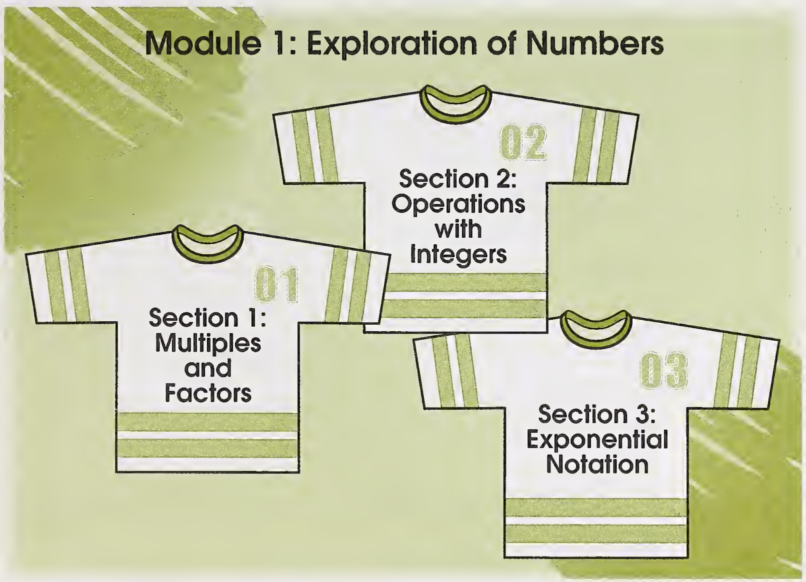
NOVA DEVELOPMENT CORPORATION

# Module Overview

Imagine a world without numbers. Could advances in science and medicine continue? Could commerce and business exist without numbers? Numbers are needed for the purposes of counting, locating, measuring, and communicating.

As a child, were you curious about numbers and their meaning? Numerals appear in many places: on football jerseys, houses, clocks, telephones, money, and so on. Over the years, your number sense and skills with numbers improved. The math courses you have taken thus far have shown you increasing complex mathematical ideas.

In this module, you will explore number concepts by working with multiples, factors, integers, and powers.

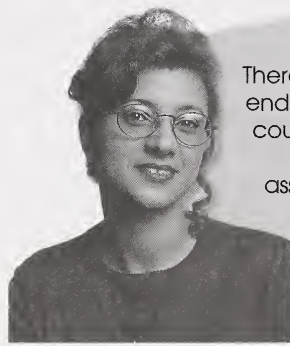



# Assessment

Your mark for this module will be determined by how well you complete the assignments at the end of each section. In this module, you must complete three assignments. The mark distribution is as follows:

| | |
|---|---|
| **Assignment Booklet 1A** | |
| **Section 1 Assignment** | **30 marks** |
| **Assignment Booklet 1B** | |
| **Section 2 Assignment** | **39 marks** |
| **Assignment Booklet 1C** | |
| **Section 3 Assignment** | **31 marks** |
| **TOTAL** | **100 marks** |

When doing the assignments, work slowly and carefully. You must do each assignment independently; but if you are having difficulties, you may review the appropriate section in this Student Module Booklet.



There is a supervised final test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

## Stragegies for Completing the Module Successfully

To achieve success in this module, be sure to work slowly and systematically through the Student Module Booklet. Remember, your work in the Student Module Booklet will prepare you for your module assignments and final test.

Following are some strategies for completing the module successfully:

- Try to set realistic goals for each day. Once you've set these goals, stick to them.

- Read all the instructions in each Student Module Booklet carefully; answer all the questions in your mathematics binder; and check your answers by comparing your responses with the suggested answers in the Appendix.

- Keep a section of your mathematics binder for your journal entries. Get in the habit of describing new concepts, procedures, and strategies in your own words. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, and charts) to help you connect mathematical ideas.

- Ask someone who is taking Mathematics Preparation 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

- If you need assistance from your teacher or your study partner, you may find it helpful to write your questions in the journal section of your mathematics binder before speaking to your teacher or study partner. This will help you pinpoint your problem.

- Take care when completing your module assignments. Be sure you have completed each part of the Assignment Booklets, and proofread the assignments before you submit them. Remember to review your module assignments after they are corrected; then file them in your binder so you can review your assignments and mathematics notes before you write the final test.

Good luck!

# SECTION 1

## Multiples and Factors

Seth and his grandfather are great buddies. They have many common interests. They enjoy rural life, attending rodeos, and riding horses. What common interests do you have with your friends and relatives?

Mathematicians, too, have common interests. They, however, are interested in what numbers have in common with each other. For example, they look for common multiples, common factors, and common denominators.

In this section, you will list the multiples of a number and find the least common multiple of two or more numbers. You will then list the factors of a number and find the greatest common factor of two or more numbers.

# Activity 1: Multiples, Common Multiples, and LCM

In elementary grades, you learned to skip count. For example, you learned to count by 3s.

$3, 6, 9, 12, \ldots$

When you skip count, what you are in fact doing is listing the multiples of 3.

**The multiples of a number are found by multiplying the number by the natural numbers or by adding the number repeatedly.**

The multiples of 3 are $\underset{3\times 1}{3}, \overset{+3}{\phantom{x}} \underset{3\times 2}{6}, \overset{+3}{\phantom{x}} \underset{3\times 3}{9}, \overset{+3}{\phantom{x}} \underset{3\times 4}{12}, \ldots$ .

1. a. List the first five multiples of 4. That is, count by 4s, beginning with 4.

   b. List the first five multiples of 5. That is, count by 5s, beginning with 5.

Check your answers by turning to the Appendix, page 113.

You can use a combination of mental math and a calculator to find the multiples of a number.

Work through the following example.

## Example

Find the first five multiples of 13 that are greater than 150.

## Solution

You can use mental math to find the first multiple of 13 that is greater than 150.

$13 \times 10 = 130$ ← This multiple is not greater than 150.

$13 \times 11 = 130 + 13$
$= 143$ ← This multiple is not greater than 150.

$13 \times 12 = 143 + 13$
$= 156$ ← This multiple is greater than 150.

You can use a calculator to determine the next four multiples of 13 that are greater than 150.

1 5 6 + 1 3 =

169.

+ 1 3 =

182.

+ 1 3 =

195.

+ 1 3 =

208.

The first five multiples are 156, 169, 182, 195, and 208.

2. For each of the following numbers, list the first 5 multiples greater than 80.

**a.** 8 **b.** 12 **c.** 23 **d.** 32

Check your answers by turning to the Appendix, page 113.

Now you will find the common multiples of two or more numbers and identify the least common multiple (LCM).

**When a number is a multiple of two or more numbers, it is called a common multiple. The least common multiple (LCM) of two or more numbers is the least of the common multiples of the numbers.**

Work through the following example.

## Example

What are the first three common multiples of 20 and 30? What is the least common multiple (LCM)?

## Solution

**Step 1:** List the multiples of 20.

20, 40, 60, 80, 100, 120, 140, 160, 180, 200, …

**Step 2:** List the multiples of 30.

30, 60, 90, 120, 150, 180, …

**Step 3:** Circle the common multiples of 20 and 30.

20, 40, 60, 80, 100, 120, 140, 160, 180, 200, …

30, 60, 90, 120, 150, 180, …

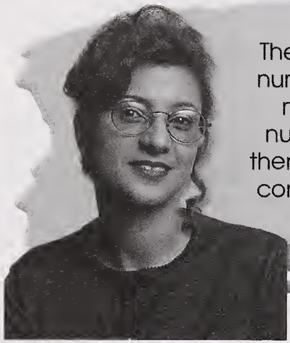
The first three common multiples are 60, 120, and 180.

**Step 4:** Put a square around the least common multiple of 20 and 30.

$20, 40, \boxed{60}, 80, 100, 120, 140, 160, 180, 200, \ldots$

$30, \boxed{60}, 90, 120, 150, 180, \ldots$

The least common multiple is 60.



There are an infinite number of common multiples of two numbers. However, there is only **one** least common multiple of two numbers.

3. **a.** List the first ten multiples of 6.
   **b.** List the first ten multiples of 9.
   **c.** Circle the common multiples of 6 and 9.
   **d.** Put a square around the least common multiple of 6 and 9.

4. Give the first three common multiples and the least common multiple for each of the following pairs of numbers:

   **a.** 4 and 6  **b.** 8 and 12  **c.** 18 and 24

5. Find the LCM for the following sets of numbers.

   **a.** 4, 6, and 10  **b.** 16, 18, and 24

6. In training for a long distance race, Camilla and Karen complete several laps of a course. Camilla's average time to complete one lap of the course is 9 min. Karen's average time to complete one lap is 12 min.

min = minutes

   **a.** If the girls begin at the same time, what is the least amount of time in which they could again be running side by side at the finish line?

   **b.** If the girls begin at the same time and they cross the finish line together, who will have completed more laps? How many more?

7. Ming waters the plants in his father's restaurant. One plant is watered every 8 days. Another plant is watered every 12 days.


a. If Ming watered both plants on December 31, what is the next date on which Ming will have to water both plants on the same day?

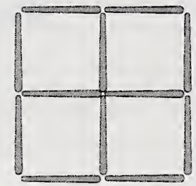
b. How many times in the year will both plants be watered on the same day?

Check your answers by turning to the Appendix, page 113.

## Now Try This

Use a problem-solving stategy to answer the following questions.

8. In the sketch, 12 toothpicks are arranged to make 4 squares of equal size. How can you make 3 squares of equal size by repositioning 3 toothpicks?
**Hint:** Each toothpick must be used as a side of a square.



9. A can of pop costs $0.80 in a vending machine. The machine accepts quarters, nickels, and dimes, but it does not give change. Make a list of all the possible combinations of coins you can use to buy a single can of pop.

10. The LCM of 8 and 5 is actually the product of 8 and 5, which is $8 \times 5 = 40$. Is the LCM of two numbers always the product of the numbers? Provide an example using two numbers where the LCM is smaller than the product of the two numbers.

Check your answers by turning to the Appendix, page 114.

The concept of least common multiple is used when you add or subtract fractions.

Fractions will be reviewed in much further depth in Module 2, but for now use your knowledge of least common multiple (LCM) to determine the least common denominator (LCD).

**The least common denominator of two or more fractions is the least common multiple of the denominators.**

## Example

Find the least common denominator of $\frac{2}{3}$ and $\frac{5}{6}$.

### Solution

List the multiples of each denominator.

3, 6, 9, 12, 15, ...

6, 12, 18, 24, 30, ...

The least common multiple of 3 and 6 is 6. Therefore, the least common denominator of the fractions is 6.

**11.** Find the least common denominator of the following pairs of fractions.

**a.** $\frac{1}{3}, \frac{4}{9}$ **b.** $\frac{5}{4}, \frac{5}{18}$ **c.** $\frac{4}{7}, \frac{1}{12}$

Check your answer by turning to the Appendix, page 115.

## Looking Back

In the first part of this activity, the main goal was to list the multiples of a given number. If the multiples were small, you used mental math to list the multiples. For larger numbers, you may have found the calculator more convenient. In the "Now Try This" section, you began developing your problem-solving skills.

**12.** In your journal, list the first nine multiples of nine, starting with 9. Do you notice any pattern when you add the digits in the ones place and the tens place of each multiple of 9? What is the pattern?

Check your answer by turning to the Appendix, page 115.

In this activity, you listed the common multiples of two or more numbers and identified the least common multiple (LCM). You also used some of the problem-solving strategies discussed in the Appendix of this module to solve problems.

# Activity 2: Multiples, Factors, and Divisibility

There are many opposite actions in life. For example, opening an umbrella is the opposite of closing an umbrella. Turning off a light is the opposite of turning on a light. Taking three eggs out of a carton is the opposite of putting three eggs into a carton.

In mathematics multiplying and dividing are opposite operations. To illustrate this, review the meaning of these operations.

**Multiplying is the process of finding the total number of elements in a given number of groups where each group has the same number of elements.**

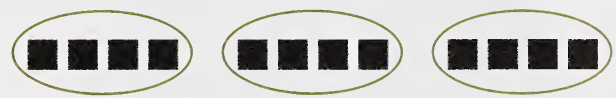
For example, to solve the multiplication problem $3 \times 4$, ask yourself, "How many elements are there in 3 groups of 4?" Of course, there are 12 elements.

**Note:** There are several mathematical terms used to describe multiplying.

$$\begin{array}{rl} 4 & \leftarrow \text{multiplicand (factor)} \\ \underline{\times\ 3} & \leftarrow \text{multiplier (factor)} \\ 12 & \leftarrow \text{product} \end{array}$$

**Dividing is the process of finding how many groups of a given size are contained in a given set of elements, or the process of finding how many elements there are in a given number of groups, where each group has the same number of elements.**

For example, to solve the division problem $12 \div 4$, ask yourself, "In 12, there are how many groups of 4?" Of course, there are 3 groups.



**Note:** There are several terms used to describe dividing.

$$\begin{array}{rl} 3 & \leftarrow \text{quotient} \\ 4\overline{)12} & \leftarrow \text{dividend} \\ \underline{12} & \\ 0 & \leftarrow \text{remainder} \end{array}$$

divisor

Because multiplication and division are opposite operations, you can use division to determine if a number is a multiple of another number.

Work through the following example.

### Example

Is 98 a multiple of 4?

### Solution

Use division to determine if 98 is a multiple of 4.

$$\begin{array}{r} 24 \\ 4\overline{)98} \\ \underline{8} \\ 18 \\ \underline{16} \\ 2 \end{array}$$

or

9 8 ÷ 4 =

24.5

There is a remainder. So, 4 does not divide evenly into 98.

The quotient is not a whole number. So, 4 does not divide evenly into 98.

No, 98 is not a multiple of 4.

1. For each of the following, determine if the first number is a multiple of the second number.

   a. 94, 7 b. 86, 4 c. 98, 12

   d. 56, 14 e. 35, 7 f. 49, 6

Check your answers by turning to the Appendix, page 115.

Because of the relationship between multiplication and division, you can use division to decide if a number is a factor of another number. Work through the following examples.

### Example

Is 4 a factor of 252?

### Solution

Use division to determine if 4 is a factor of 252.

$$\begin{array}{r} 63 \\ 4\overline{)252} \\ \underline{24} \\ 12 \\ \underline{12} \\ 0 \end{array}$$

or

2 5 2 ÷ 4 =

63.

There is no remainder. So, 4 divides evenly into 252.

The quotient is a whole number. So, 4 divides evenly into 252.

Yes, 4 is a factor of 252.

## Example

There are 676 students in Cobequid High School. The principal wishes to group the students into teams for intramural sports. Each team must have the same number of students. Can the teams have exactly 9 students each?

### Solution

Use division to determine if 9 is a factor of 676.

$$\begin{array}{r} 75 \\ 9\overline{)676} \\ \underline{63}\phantom{6} \\ 46 \\ \underline{45} \\ 1 \end{array}$$

or

6 7 6 ÷ 9 =

75.11111111

There is a remainder.

The quotient is not a whole number.

The teams cannot have exactly 9 students each because 9 is not a factor of 676.

2. Is 3 a factor of each of the following numbers?

   **a.** 1306 **b.** 9858 **c.** 5842

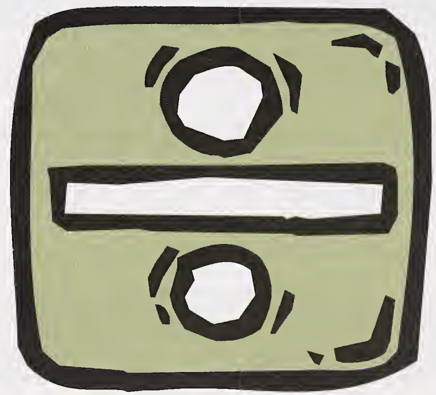
3. Is 6 a factor of each of the following numbers? Answer **yes** or **no**.

   **a.** 1215 **b.** 8743 **c.** 5394

Check your answers by turning to the Appendix, page 116.

## Divisibility Tests

Thus far, you determined whether a number is a factor of another number using division, either by performing long division or by using your calculator. It is also possible to determine whether a number is a factor of another number by simply using divisibility tests. You may recall these tests from previous mathematics courses.



The following table reviews each divisibility test.

### Divisibility Tests

| Factor | Condition | Example |
|---|---|---|
| 2 | The number is even. | 6934 is an even number; so, 6934 is divisible by 2. |
| 3 | The sum of the digits is divisible by 3. | $8+7+5+4$ is divisible by 3; so, 8754 is divisible by 3. |
| 4 | The last two digits are divisible by 4. | 24 is divisible by 4; so, 17 324 is divisible by 4. |
| 5 | The last digit is 0 or 5. | The last digit is 5; so, 17 365 is divisible by 5. |
| 6 | The number is divisible by both 2 and 3. | 2382 is even and $2+3+8+2$ is divisible by 3; so, 2382 is divisible by 6. |
| 8 | The last three digits are divisible by 8. | 128 is divisible by 8; so, 159 128 is divisible by 8. |
| 9 | The sum of the digits is divisible by 9. | $8+6+4$ is divisible by 9; so, 864 is divisible by 9. |
| 10 | The last digit is 0. | The last digit is 0; so, 320 is divisible by 10. |

Use divisibility tests to answer questions 4 to 11.

**4.** Does each of these numbers have 2 as a factor? Answer **yes** or **no**.

**a.** 13 582 **b.** 19 876 **c.** 45 769

**5.** Does each of these numbers have 5 as a factor? Answer **yes** or **no**.

**a.** 19 420 **b.** 56 005 **c.** 875 329

**6.** Does each of these numbers have 10 as a factor? Answer **yes** or **no**.

**a.** 92 533 **b.** 199 345 **c.** 18 650

**7.** Does each of these numbers have 4 as a factor? Answer **yes** or **no**.

**a.** 7168 **b.** 3354 **c.** 6528
**d.** 2616 **e.** 35 638 **f.** 516 082

**8.** Does each of these numbers have 8 as a factor? Answer **yes** or **no**.

**a.** 191 080 **b.** 37 206 **c.** 68 592
**d.** 70 034 **e.** 18 320 **f.** 365 168

**9.** Does each of these numbers have 3 as a factor? Answer **yes** or **no**.

**a.** 6313 **b.** 62 160 **c.** 5241

**10.** Does each of these numbers have 6 as a factor? Answer **yes** or **no**.

**a.** 28 312 **b.** 37 032 **c.** 37 293

**11.** Do each of these numbers have 9 as a factor? Answer **yes** or **no**.

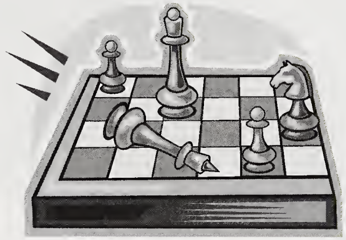
**a.** 70 038 **b.** 27 099 **c.** 63 012

Check your answers by turning to the Appendix, page 116.

## Now Try This

Use a problem-solving strategy to answer the following question.

**12.** There are 8 people involved in a chess tournament. The organizers decide that each participant will play everyone else one game of chess, and the person who wins the most games will be the winner. How many games of chess will be played in the tournament?



Check your answer by turning to the Appendix, page 117.

## Looking Back

In this activity, you discovered the relationship between multiples and factors and determined whether a number is a factor of another number by using division or by using the divisibility tests.

**13.** In your journal, write a number greater than 1000 and less than 2000 that is divisible by 4. Use your knowledge of the divisibility test for 4 to determine your number. Then verify that your number is divisible by 4 by performing long division. List at least two factors of your number.

Check your answer by turning to the Appendix, page 117.

# Activity 3: Factors, Common Factors, and GCF

When you go shopping, do you make a list? Making a list is helpful if you do not want to forget anything.

In this activity, you will list all the factors of a number.

One way to list all the factors of a number, making sure not to miss any, is to begin with the number 1 and test each consecutive number to see if it is a factor.

**Consecutive numbers are integers that increase in order by 1. For example, 1, 2, 3, and 4 are consecutive numbers.**

You may find it helpful to organize the numbers in a T-table and put a stroke through the numbers that are not factors.

**A T-table is a table made in the shape of the letter T.**

Work through the following example.

## Example

List all the factors of 48.

## Solution

| | 48 | |
|---|---|---|
| 1 | 48 | |
| 2 | 24 | |
| 3 | 16 | |
| 4 | 12 | |
| ~~5~~ | | ← 5 is not a factor. |
| 6 | 8 | |
| ~~7~~ | | ← 7 is not a factor. |
| ~~8~~ | ~~6~~ | ← 8 and 6 have been used already. |

To list the factors of 48, read down the left-hand side of the T-table and up the right-hand side.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

**1.** Use T-tables to list all the factors of each of the following numbers.

**a.** 30 **b.** 32 **c.** 45 **d.** 70

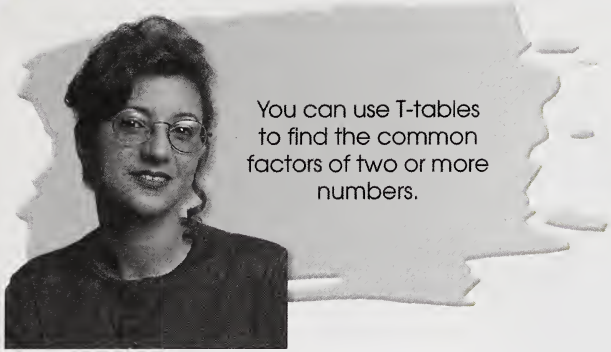
**2.** Mike likes to collect insects. He wants to arrange 12 butterflies in rows with the same number of butterflies in each row. What are the different possible combinations? **Hint:** Find the factors of 12.

Check your answers by turning to the Appendix, page 117.

## Common Factors

In this part of the activity, you will list the common factors of two or more numbers.

**A common factor is a factor that is common to two or more numbers.**




Work through the following example.

## Example

List the common factors of 24 and 30.

### Solution

**Step 1:** List all the factors of 24 and all the factors of 30.

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

**Step 2:** Circle the common factors of 24 and 30. That is, circle the numbers that are in both T-tables.

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

The common factors of 24 and 30 are 1, 2, 3, and 6.

**3.** Use T-tables to list the common factors for each pair of numbers.

**a.** 24 and 36 **b.** 12 and 18 **c.** 30 and 75

Check your answers by turning to the Appendix, page 118.

## Greatest Common Factor

Often, it is useful to find the greatest common factor (GCF) of two or more numbers.

**The greatest common factor is the greatest factor of two or more numbers.**

## Example

A grocer wants to make up gift packages that contain two kinds of candy. Each package must have the same number of each type of candy. One type of candy comes in original boxes that contain 18 pieces, and the other type comes in bags that contain 24 pieces. What is the greatest number of gift packages the grocer can make from a box of one type of candy and a bag of the other type?

### Solution

**Step 1:** List all the factors of 18 and all the factors of 24.

| 18 | |
|---|---|
| 1 | 18 |
| 2 | 9 |
| 3 | 6 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | ~~3~~ |

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

**Step 2:** Circle the common factors of 18 and 24.

| 18 | |
|---|---|
| 1 | 18 |
| 2 | 9 |
| 3 | 6 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | ~~3~~ |

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

The common factors of 18 and 24 are 1, 2, 3, and 6.

**Step 3:** Draw a square around the greatest number in the list of common factors.

| 18 | |
|---|---|
| 1 | 18 |
| 2 | 9 |
| 3 | $\boxed{6}$ |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | ~~3~~ |

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | $\boxed{6}$ |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

So, the greatest number of gift packages the grocer can make from one box and one bag is 6.

**4.** What is the greatest common factor for each of these pairs of numbers?

**a.** 6 and 12 **b.** 30 and 45 **c.** 42 and 56

**5.** What is the greatest common factor of each of these sets of numbers?

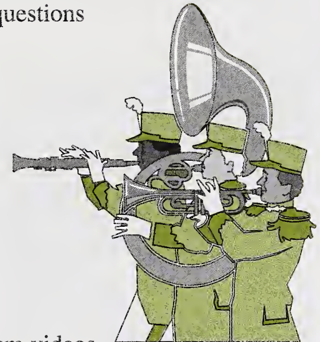
**a.** 54, 63, and 81 **b.** 8, 16, and 20 **c.** 24, 42, and 48

**6.** Jeanine, Julie, and Joyce bought some candy at the same store. Jeanine bought 28¢ worth, Julie bought 72¢ worth, and Joyce bought 96¢ worth. What is the most that each piece of candy could have cost?

Check your answers by turning to the Appendix, page 119.

## Now Try This

You are now ready for some more challenging questions on LCM.



7. The members of a marching band tried to arrange themselves in rows of 2, 3, 4, or 5 with exactly the same number of people in each row, but each time there was one person left over. What is the least number of members the marching band could have had?

8. An information centre is showing two tourism videos. The first video is 36 min long; the second video is 45 min long. If each video begins at 8:45 A.M. and runs continuously until 5:45 P.M., at what time will both start together again?

Check your answers by turning to the Appendix, page 120.

## Looking Back

In this activity, you listed the factors of a number and found the greatest common factor (GCF) of two or more numbers.

The concept of greatest common factor is used when you are simplifying fractions. Fractions will be reviewed in further detail in Module 2; but for now, you will use your knowledge of greatest common factor to simplify a fraction.

Work through the following example.

### Example

Simplify $\frac{18}{24}$.

### Solution

**Step 1:** Find the greatest common factor of 18 and 24.

| 18 | | | 24 | |
|---|---|---|---|---|
| 1 | 18 | | 1 | 24 |
| 2 | 9 | | 2 | 12 |
| 3 | $\boxed{6}$ | | 3 | 8 |
| $\cancel{4}$ | | | 4 | $\boxed{6}$ |
| $\cancel{5}$ | | | 5 | |
| $\cancel{6}$ | $\cancel{3}$ | | $\cancel{6}$ | $\cancel{4}$ |

The greatest common factor is 6.

**Step 2:** Simplify $\frac{18}{24}$ by dividing the numerator and denominator by the greatest common factor, 6.

$$\frac{18}{24} \overset{\div 6}{\underset{\div 6}{=}} \frac{3}{4}$$

Therefore, the simplified form of $\frac{18}{24}$ is $\frac{3}{4}$.

9. In your journal, simplify the fraction $\frac{36}{81}$ by dividing both the numerator and denominator by the greatest common factor.

Check your answer by turning to the Appendix, page 120.

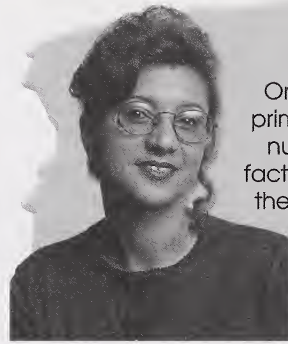
## Activity 4: Another Way to Find the LCM and GCF

Michelle and Vern are chefs. Michelle likes to fry hamburgers, and Vern likes to barbecue them. There is often more than one way to accomplish a task.

Earlier in this module, you found the least common multiple (LCM) of two or more numbers. You listed the multiples of each number, circled the common multiples, and chose the least of the common multiples. You then found the greatest common factor (GCF) of two or more numbers using a similar method. You listed the factors of each number, circled the common factors, and chose the greatest of these common factors.

In this activity, you will discover another way of determining the LCM and GCF of two or more numbers. This new method is called prime factorization.

**Prime factorization is the process of finding all the prime factors of a given number. Each factor is only divisible by itself or 1.**



One way to write the prime factorization of a number is to make a factor tree. Work through the following example.

### Example

Write 36 as a prime factorization.

### Solution

$$\begin{array}{c} 36 \\ 6\times 6 \\ 2\times 3\times 2\times 3 \end{array} \quad \text{or} \quad \begin{array}{c} 36 \\ 4\times 9 \\ 2\times 2\times 3\times 3 \end{array}$$

So, $36 = 2\times 2\times 3\times 3$.

**Note:** It is customary to arrange the prime factors from least to greatest.

1. Find the prime factorization of each of these numbers.

**a.** 38 **b.** 42 **c.** 45

Check your answers by turning to the Appendix, page 120.

## Finding the GCF of Two or More Numbers

You can find the GCF of two or more numbers by writing the prime factorizations of each of the numbers, arranging the prime factors so that the common prime factors are aligned in columns, taking one factor from each complete column, and multiplying these factors.

This is illustrated in the following example.

### Example

Robin paid $78 for several doorknob sets. Eddie bought several doorknob sets for $117. If all the doorknob sets were the same price, what is the most that each doorknob set could have cost?

### Solution

**Step 1:** Use factor trees to write the prime factorization of each number.

$$117 \to 3\times 39 \to 3\times 3\times 13 \qquad 78 \to 2\times 39 \to 2\times 3\times 13$$

**Step 2:** Arrange the prime factorization of each number so the common prime factors are aligned in columns.

$$\begin{array}{rcccc} 117 = & & \boxed{3} & \times 3\times & \boxed{13} \\ 78 = & 2\times & \boxed{3} & \times & \boxed{13} \end{array}$$

The common prime factors are 3 and 13.

**Step 3:** To find the GCF, take one factor from each **complete** column and multiply.

$$\begin{aligned} GCF &= 3\times 13 \\ &= 39 \end{aligned}$$

So, the most each doorknob set could cost is $39.

2. Find the GCF of each set of numbers.

**a.** 24 and 36 **b.** 12 and 18 **c.** 20, 30, and 75

Check your answers by turning to the Appendix, page 121.

## Finding the LCM of Two or More Numbers

You can find the LCM of two or more numbers by writing the prime factorization of the numbers, arranging the prime factors so that the common prime factors are aligned in columns, taking a factor from each column, and multiplying these factors.

Work through the following example.

## Example

A surveyor begins putting white and green pegs into the ground. Starting from a particular tree, he puts a white peg every 30 m and a green peg every 50 m. How far from the tree will the surveyor be when he first puts both a white peg and a green peg into the ground?

### Solution

**Step 1:** Write the prime factorization of each number.

$$30 = 6 \times 5 = 2 \times 3 \times 5$$

$$50 = 2 \times 25 = 2 \times 5 \times 5$$

**Step 2:** Arrange the prime factorizations so the common prime factors are aligned in columns.

| | | | | |
|---|---|---|---|---|
| 30 = | 2 | × 3 | × 5 | |
| 50 = | 2 | | × 5 | × 5 |

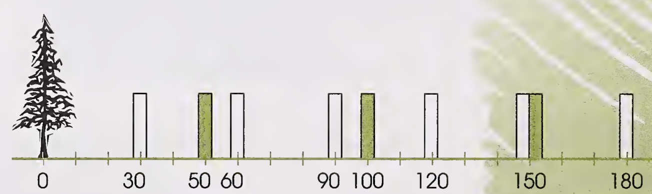
**Step 3:** To find the LCM, take one prime factor from each column and multiply.

$$\begin{aligned} LCM &= 2 \times 3 \times 5 \times 5 \\ &= 150 \end{aligned}$$

The LCM is 150.

So, the surveyor will be 150 m from the tree when he first puts in both a white peg and a green peg.

**Note:** This diagram may help you visualize the situation.




3. Use your factor trees from question 2 to find the LCM for each set of numbers.

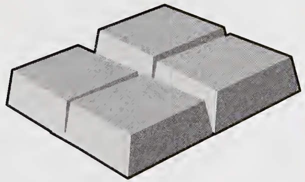
   **a.** 24 and 36 **b.** 12 and 18 **c.** 20, 30, and 75

Check your answers by turning to the Appendix, page 121.

## Now Try This

Use a problem-solving strategy to answer the following question.

**4.** A piece of chocolate costs \$0.30, \$0.40, or \$0.60, depending on the type you buy. Samantha bought the same number of \$0.40 chocolate as \$0.60 chocolate. She also bought some \$0.30 chocolate. If Samantha paid \$6.90 for 15 pieces of chocolate, how many pieces of \$0.30 chocolate did she buy?



Check your answer by turning to the Appendix, page 122.

## Looking Back

In this activity, you used the prime factorizations of two or more numbers to determine the greatest common factor and the least common multiple. You used a different method to find the LCM and GCF earlier in the module.

**5.** In your journal, use two methods to find the LCM of 30 and 42.

**6.** In your journal, use two methods to find the GCF of 30 and 42.

Check your answers by turning to the Appendix, page 123.

## Conclusion

In this section, you listed the multiples of a number and found the least common multiple of two or more numbers. You then listed the factors of a number and found the greatest common factor of two or more numbers.

Mathematicians are not the only ones who work with these common elements. In this section, you saw many ways in which the ideas of common multiples and common factors were used to solve everyday problems.

Everyone looks for things they have in common with others. Frank and Denny are best friends. They enjoy playing football and video games together. What common interests do you share with your friends?

## Assignment

Turn to Assignment Booklet 1A and complete the assignment for Section 1.

# SECTION 2

## Operations with Integers

Hockey is a very popular sport in North America. It is played at all levels of expertise, from the local skating rink with neighbourhood friends to international levels to professional leagues (where players make hockey their career).

In competitive hockey, statistics are kept on the number of goals players score and the number of assists they make. These records are used by players and coaches to improve the quality of play and the overall success of the team. Because each player does not have the same opportunity to score or assist on goals, a plus-minus system is used. Players, except the goalie, are credited with a plus (+) each time they are on the ice when their team scores. Players on the ice when the opposition scores a goal are credited with a minus (–). Each player is rated over a number of games by adding the points for and the points against.

In this section, you will use learning aids to visualize the operations of addition, subtraction, multiplication, and division with integers. You will solve problems with integers and use the rules for order of operations to evaluate expressions with integers.

# Activity 1: Exploring Integers

Do you like being outdoors in cold weather, or would you rather be curled up in front of a fire when the temperature is $-20°C$?

You know that room temperature is $+20°C$, the freezing point of water is 0°C, and $-20°C$ is considered to be a cold day. Did you know that $+20$, 0, and $-20$ are integers?

**The set of integers includes these numbers:**

$$\{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$$

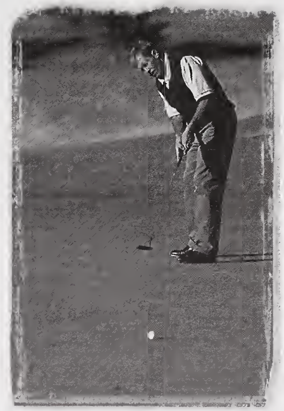
Zero acts like a reference point in the set of integers.

- The positive integers are greater than zero. To indicate this, a plus sign (+) is sometimes written before the numeral.
- The negative integers are less than zero. A negative integer is always written with a minus sign (–) before the numeral.

In questions 1 to 4, you will discover that many situations can be represented by integers.

1. In some hotels, parking levels are below ground, the lobby is on the ground level, and the guest rooms are on the floors above the lobby. What integer could be used to indicate each level on this hotel elevator panel?


2. In the game of golf, a player tries to get the ball into each of the holes on the golf course in as few strokes as possible.



The standard number of strokes set for each hole is called "par." Scores can be expressed as integers with par being zero. What integer could be used to indicate each of the following scores?

**a.** two under par
**b.** five over par

3. Accountants use the phrases "in the black" or "in the red" to describe a company's financial position. "In the red" means the company is in debt; it is losing money. "In the black" means the company is making a profit. What integer could be used to indicate the financial position described in each of the following situations?

**a.** a profit of $5000
**b.** a loss of $800
**c.** a surplus of $20
**d.** a withdrawal of $75
**e.** no change in the balance
**f.** a deposit of $100
**g.** a deficiency of $45

4. The following table lists the high and low temperatures for several cities in Canada on a particular day in March.

| City | High (°C) | Low (°C) |
|---|---|---|
| Calgary | 13 | −2 |
| Charlottetown | −7 | −15 |
| Edmonton | 8 | −5 |
| Fredericton | −5 | −14 |
| Halifax | −6 | −13 |
| Inuvik | −26 | −36 |
| Ottawa | −4 | −6 |
| Prince George | 9 | 0 |
| Saskatoon | 6 | −4 |
| St. John's | −3 | −11 |
| Vancouver | 11 | 5 |
| Whitehorse | −11 | −18 |
| Winnipeg | 0 | −5 |
| Yellowknife | −11 | −14 |

**a.** What is the lowest recorded temperature on the table? For which city?
**b.** What is the highest recorded temperature on the table? For which city?
**c.** Which city was warmer on that day, Charlottetown or Halifax?
**d.** Which city was colder on that day, Charlottetown or Fredericton?

Check your answers by turning to the Appendix, page 123.

## Absolute Value

Every positive integer and negative integer has a sign and an absolute value.

**The absolute value of an integer is the value of the integer without regard to the sign. The symbol for absolute value is two vertical line segments. Sometimes the absolute value of an integer is called its magnitude.**

### Example

A cold winter day is $-20°C$. Room temperature is $+20°C$. What is the absolute value of $-20$ and $+20$?

### Solution

$|-20| = 20$

$|-20|$ is read as "the absolute value of negative 20."

$|+20| = 20$

$|+20|$ is read as "the absolute value of positive 20."

The absolute value of $-20$ is 20. The absolute value of $+20$ is 20.

**Note:** Both $+20°C$ and $-20°C$ are 20° from the freezing point. However, $+20°C$ is 20° above the freezing point and $-20°C$ is 20° below the freezing point.

The integers $+20$ and $-20$, as worked with in the example, are opposite integers.

**Two integers are opposite integers if they have the same absolute value but different signs. Sometimes the opposite integer is called the additive inverse.**

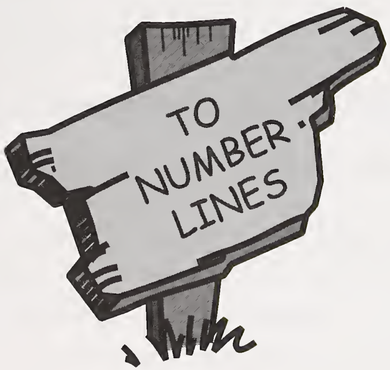
**5.** Give the absolute value of each of the following.

**a.** $-3$ **b.** $+7$ **c.** $+5$ **d.** $-4$

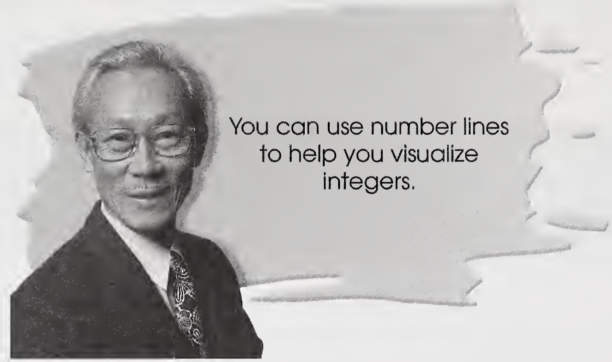
**6.** Give the opposite integers (additive inverse) of each of the following.

**a.** $-2$ **b.** $+8$ **c.** $+6$ **d.** $-3$

Check your answers by turning to the Appendix, page 124.




## Number Lines




The following is a horizontal number line. It is used to compare and order integers.

$$\longleftarrow -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4 \longrightarrow$$

The numbers on a horizontal line increase as you move from left to right.

- $+4$ is to the right of $+1$; so, $+4 > +1$.
- $+1$ is to the right of $-2$; so, $+1 > -2$.
- $-1$ is to the right of $-4$; so, $-1 > -4$.

The numbers decrease as you move from right to left.

- $-4$ is to the left of $-1$; so, $-4 < -1$.
- $-2$ is to the left of $+1$; so, $-2 < +1$.
- $+1$ is to the left of $+4$; so, $+1 < +4$.

7. Complete each of the following number lines.

a. ← ___ ___ −1 0 +1 ___ ___ ___ ___ →

b. ← ___ ___ ___ ___ ___ ___ −4 ___ ___ →

8. Use < or > to show the relationship between each pair of integers.

a. 6 ○ 16 b. −5 ○ −3 c. −2 ○ −20

d. −12 ○ 1 e. +4 ○ +5 f. 0 ○ −3

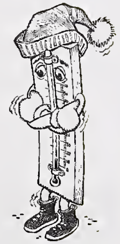
9. Arrange each group of integers in order from least to greatest.

a. −4, 8, −3, 12, 0

b. +6, 0, −4, −8, +11, +4, −10, −5

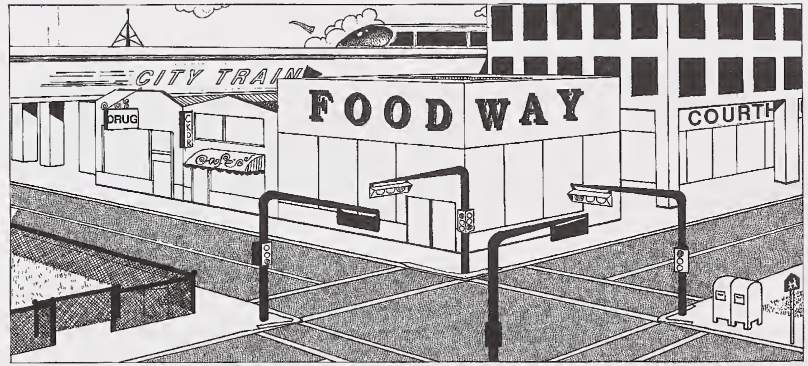
c. −1, −3, −6, +8, +11, +13, −15

10. Cyril watches the weather report one day and finds that the daily high for Winnipeg was −18°C and the daily high for Edmonton was −12°C. Because −18°C is colder than −12°C, he concludes that the integer −18 is greater than the integer −12. Is Cyril's conclusion true? Explain.



Check your answers by turning to the Appendix, page 124.

## Cartesian Coordinate System




In many western communities, avenues run east and west and streets run north and south. The streets and avenues create a grid. The location of a building on this grid is described by two numbers: the building number and the street or avenue number.

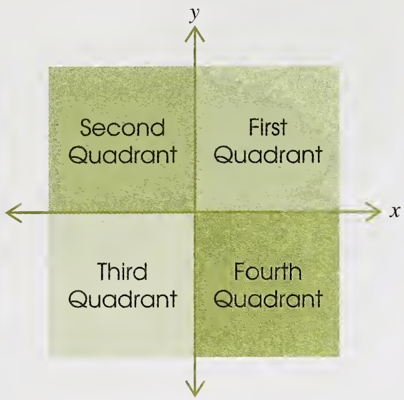
In the Cartesian coordinate system, the horizontal axis ($x$-axis) is used to describe movements left or right of the **origin**. The vertical axis ($y$-axis) is used to describe movements up or down from the origin.

**The Cartesian coordinate system is a method of defining the position of a point in two-dimensional space. The origin, $(0, 0)$, is the point where the $x$-axis and the $y$-axis meet.**

The coordinates of a point are given by two numbers, called an ordered pair. They show the position of a point with respect to the $x$-axis and the $y$-axis. The first number of the pair represents the position left or right of the origin. The second number represents the position above or below the origin.

Points may be located on the $x$-axis, on the $y$-axis, or in any of the four quadrants.

**A quadrant is one of the four regions formed by the intersection of the $x$- and $y$-axes. Quadrants are numbered counterclockwise, starting in the upper right-hand region.**




## Example

Plot the points $(-5, 2)$, $(2, -3)$, and $(-4, -1)$.

## Solution

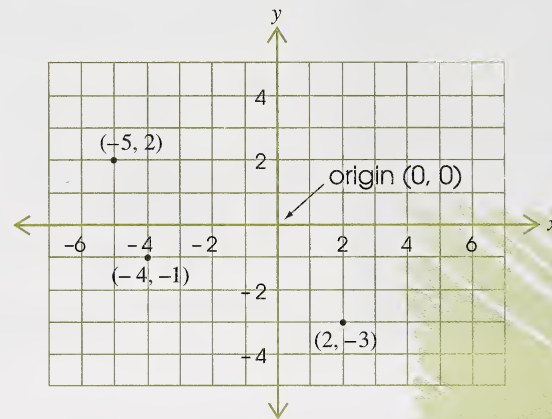
Your graph will show these points in the Cartesian coordinate system.

**Step 1:** Draw and label the $x$-axis and $y$-axis on a piece of graph paper.

**Step 2:** To plot the point $(-5, 2)$, begin at the origin. Count 5 units to the left and then 2 units up. Make a dot and label it $(-5, 2)$.
**Note:** This point is in the second quadrant.

**Step 3:** To plot the point $(2, -3)$, begin at the origin. Count 2 units to the right and then 3 units down. Make a dot and label it $(2, -3)$.
**Note:** This point is in the fourth quadrant.

**Step 4:** To plot the point $(-4, -1)$, begin at the origin. Count 4 units to the left and 1 unit down. Make a dot and label it $(-4, -1)$.
**Note:** This point is in the third quadrant.



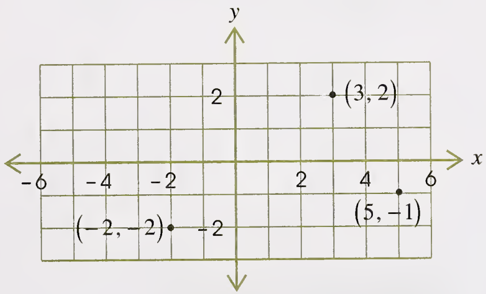

Geometric figures can be represented in the Cartesian system.

## Example

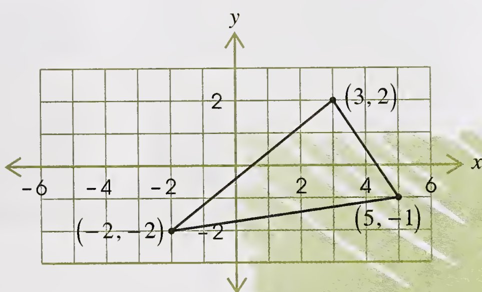
Plot the points $(3, 2)$, $(5, -1)$, and $(-2, -2)$; then connect the points. What figure is created?

## Solution

**Step 1:** Plot the points and label them.




**Step 2:** Connect the points.




In this case, a triangle is formed.

11. Plot the points $(4, 1)$, $(-1, 1)$, $(-1, -4)$, and $(4, -4)$; then connect the points. What figure is created?

12. In the Appendix, either photocopy or pull out the sheet entitled "Puzzle"; then complete the puzzle.

Check your answers by turning to the Appendix, page 124.

## Looking Back

In this activity, you discovered that integers have both magnitude and direction. You represented situations with integers and compared and ordered integers. You then used number lines and the Cartesian Coordinate system.

13. In your journal, give examples of everyday situations where integers are used.

Check your answer by turning to the Appendix, page 125.

# Activity 2: Adding and Subtracting Integers

Have you ever noticed that electrical wires are different colours? Electricians use the colours to determine which wires need to be connected.

There are two kinds of electrical charges: positive charges and negative charges. If a positive charge comes together with a negative charge, the result is no charge.

As you recall from the previous activity, an integer is either positive (+), negative (–), or 0. Two colours of counters can be used to model integers.

(+) represents $+1$

(−) represents $-1$

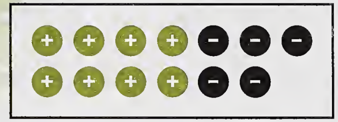
(+) (−) represents 0

Notice that combining two different colours of counters creates a **zero pair**.

Whenever you have a combination of positive and negative counters, you should rearrange the counters to make as many zero pairs as possible. Then you should consider the surplus counters to determine what integer is being represented.

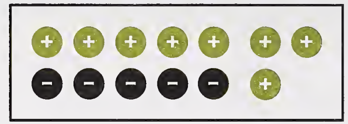
## Example

What integer is being represented by the following counters?

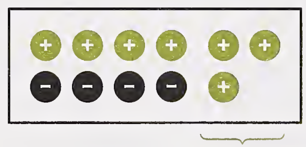


## Solution

**Step 1:** Rearrange the counters, creating as many zero pairs as possible.



**Step 2:** To determine the integer being represented, consider the surplus counters. Are the surplus counters positive or negative? How many are there?



surplus counters

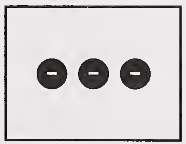
There is a surplus of **three positive counters.**

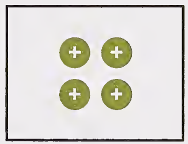
The integer being represented is $+3$.

At the back of the Appendix, find the page of positive counters and the page of negative counters. Photocopy these pages. You may wish to laminate the photocopied pages or glue them to heavier paper or boxboard (cereal boxes) before cutting out the counters. This will make the counters easier to handle.

If you prefer, you can use checkers, bingo chips, or even two kinds of coins.

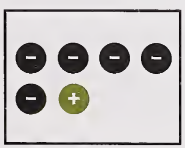
1. State the integers modelled in each of the following diagrams.

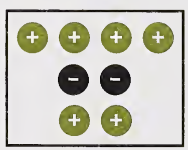
**a.** 

**b.** 

**c.** 

**d.** 

**e.** 

**f.** 

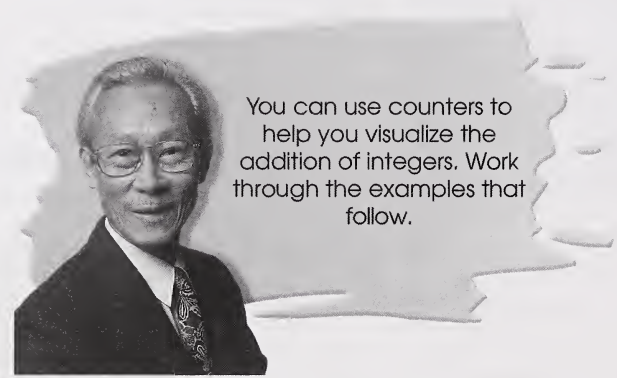
2. Using your counters, build three different models of each of the following integers.

**a.** 0 **b.** +1 **c.** −3

Check your answers by turning to the Appendix, page 125.

**Note:** Save the counters. You will need them several times throughout this module.

## Adding Integers



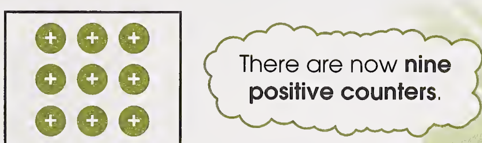
## Example

Use modelling to evaluate the expression $(+3)+(+6)$. This expression may also be written as $3+6$.

## Solution

**Step 1:** Model the expression. Start with three positive counters to represent $+3$; then add six more positive counters to represent $+6$.


**Step 2:** Find the sum. **Hint:** Are the counters negative or positive? How many are there?



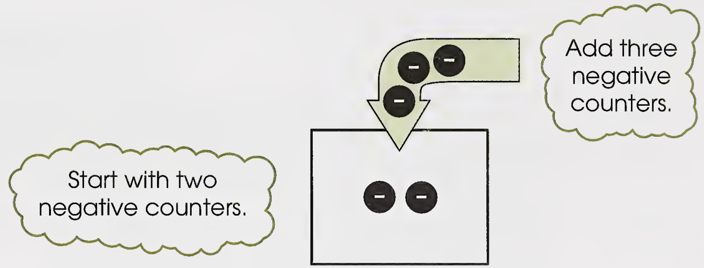

$\therefore\ (+3)+(+6)=+9$

## Example

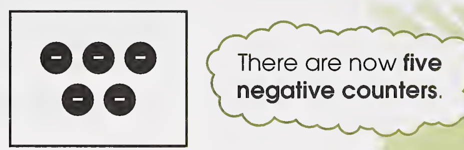
Use modelling to evaluate the expression $(-2)+(-3)$. This expression may also be written as $-2+(-3)$. **Note:** Parentheses are used to separate the sign of the number and the operation sign.

## Solution

**Step 1:** Model the expression. Start with two negative counters; then add three negative counters.




**Step 2:** Find the sum. **Hint:** Are the counters negative or positive? How many are there?



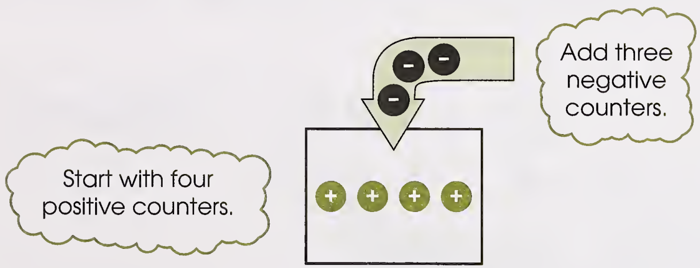

$\therefore\ (-2)+(-3)=-5$

## Example

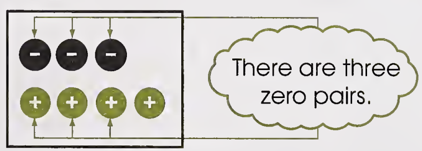
Use modelling to evaluate the expression $(+4)+(-3)$. This expression may also be written as $4+(-3)$.

### Solution

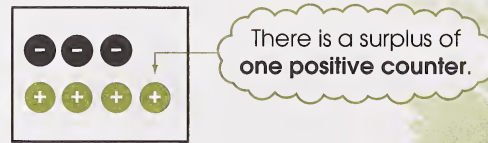
**Step 1:** Model the expression. Start with four positive integers; then add three negative integers.




**Step 2:** Because there is a combination of positive and negative counters, rearrange the counters to make as many zero pairs as possible.




**Step 3:** Find the sum. **Hint:** Are the surplus counters positive or negative? How many are there?



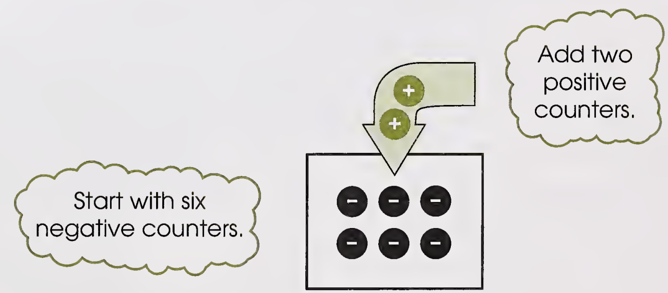

$\therefore (+4)+(-3)=+1$

## Example

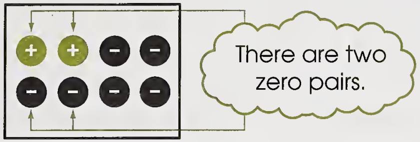
Use modelling to evaluate the expression $(-6)+(+2)$. This expression may also be written as $-6+2$.

### Solution

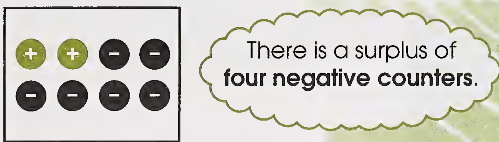
**Step 1:** Model the expression. Start with six negative counters; then add two positive counters.



**Step 2:** Because there is a combination of positive and negative counters in the container, rearrange the counters to make as many zero pairs as possible.




**Step 3:** Find the sum. **Hint:** Are the surplus counters in the container positive or negative? How many are there?




$\therefore\ (-6)+(+2)=-4$

3. Model each of these sums; then write the answer statements.

**a.** $(+2)+(+5)$

**b.** $(-3)+(+8)$

**c.** $\begin{array}{r} (-5) \\ +(-4) \\ \hline \end{array}$

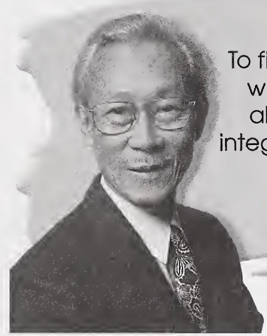
**d.** $\begin{array}{r} (+4) \\ +(-9) \\ \hline \end{array}$

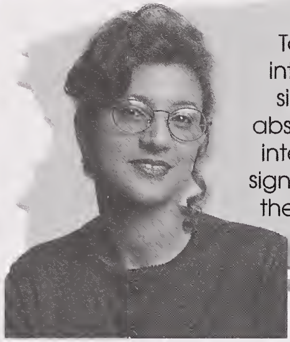
Check your answers by turning to the Appendix, page 126.

Through your work with counters, you discovered the following:

- If the signs of the integers being added are the **same**, add only one type of counter. Obviously, you will end up with more of that type of counter.
- If the signs of the integers are **different**, add some positive counters and some negative counters. After making as many zero pairs as possible, only one type of counter remains. The type of counter that remains is whichever type there was more of before making the zero pairs.

Your work with counters should have helped you to generalize two rules about adding integers.





To find the sum of integers with **unlike** signs, subtract the absolute values of the integers and use the sign of the integer with the greater absolute value.

**4.** Mentally compute each of these sums.

**a.** $(-3)+(-5)$ **b.** $(-1)+(+6)$ **c.** $(-6)+(+8)$

**d.** $\begin{array}{r}(+5)\\ +(-5)\\ \hline\end{array}$ **e.** $\begin{array}{r}(-5)\\ +(+3)\\ \hline\end{array}$ **f.** $\begin{array}{r}(+1)\\ +(+4)\\ \hline\end{array}$

**g.** $-7+5$ **h.** $-4+(-3)$

Check your answers by turning to the Appendix, page 128.

You may find a real-world application helpful in visualizing the addition of integers.

Work through the following example.

### Example

In a game of golf, Jenny scored 2 over par on the first hole and 1 under par on the second hole. What was her score after two holes?

### Solution

To solve this problem, you can write a mathematical expression involving the addition of integers.

$$\underbrace{(+2)}_{\text{first score}}+\underbrace{(-1)}_{\text{second score}}=+1$$

Jenny's score after two holes was 1 over par.

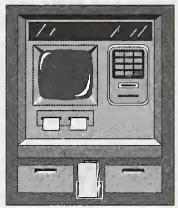
5. Write a mathematical expression to describe each of the following situations, and calculate each answer.

   a. Franz increased the speed of his car by 45 km/h. Then he decreased the speed 10 km/h. How much did he increase or decrease the speed of the car altogether?

   b. An airplane descends 400 m and then rises 100 m. How much did the airplane descend or rise altogether?

   c. The temperature fell 3°C in one hour and then fell a further 4°C in the second hour. How much did the temperature rise or fall altogether?

   d. Jasmine made a bill payment of $50 out of her bank account. Then she withdrew $40 from her bank account. How much did Jasmine's account increase or decrease altogether?



   e. Frank deposited $420 into his bank account and then withdrew $100. How much did his bank account balance rise or fall altogether?

Check your answers by turning to the Appendix, page 128.

## Subtraction of Integers

You can use counters to help you visualize the subtraction of integers.

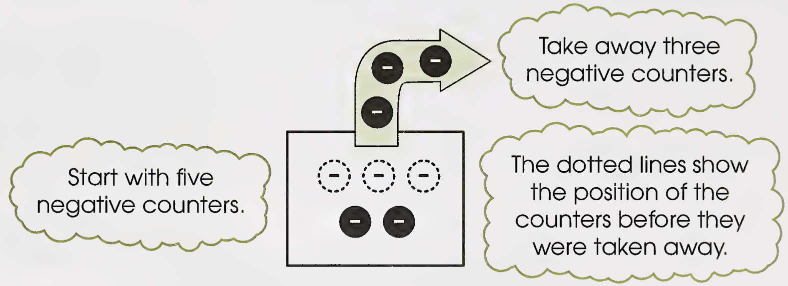
Work through the following examples.

### Example

Use modelling to evaluate the expression $(-5)-(-3)$. This expression may also be written as $-5-(-3)$.

### Solution

**Step 1:** Model the expression. Start with five negative counters to represent $(-5)$; then take away three negative counters to represent $-(-3)$.




**Step 2:** Find the difference. **Hint:** Are the remaining counters negative or positive? How many counters are left?

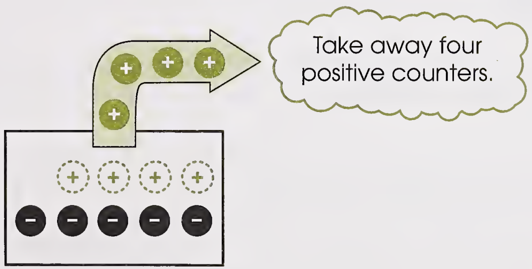

$\therefore\ (-5)-(-3)=-2$

## Example

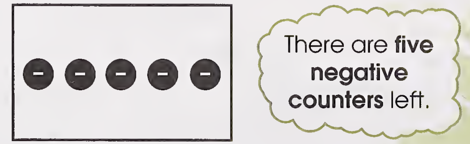
Use modelling to evaluate the expression $(-1)-(+4)$. This expression may also be written as $-1-4$.

## Solution

**Step 1:** Model the expression. Start with one negative counter. Initially, four positive counters cannot be taken away. However, if you add four zero pairs, you will not change the value of the expression because it is like adding 0 overall. Then you will have the four positive counters to take away.


**Step 2:** Find the difference. **Hint:** Are the remaining counters negative or positive? How many counters are left?



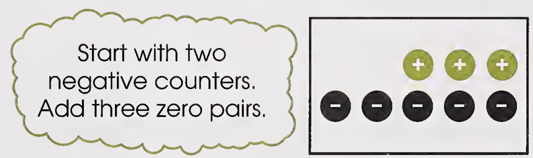

$\therefore\ (-1)-(+4)=-5$

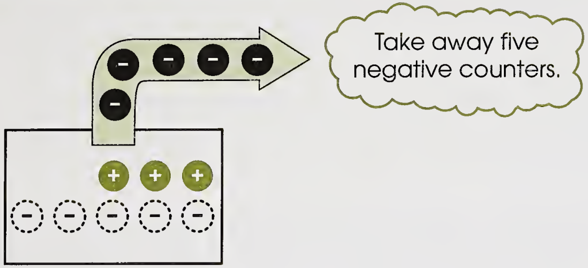
## Example

Use modelling to evaluate the expression $(-2)-(-5)$. This expression may also be written as $-2-(-5)$.

## Solution

**Step 1:** Model the expression. Start with two negative counters. Initially, five negative counters cannot be taken away. However, if you add three zero pairs, you will not change the value of the expression, but you will be able to take away five negative counters.






**Step 2:** Find the difference. **Hint:** Are the remaining counters negative or positive? How many counters are left?


$\therefore\ (-2)-(-5)=+3$

Now that you can model the subtraction of integers, do questions 6 and 7.

**6.** **a.** Model the expression $(-4)-(-2)$; then find the difference.

**b.** Model the expression $(-4)+(+2)$; then find the sum.

**c.** What do you notice about the value of expression $(-4)-(-2)$ and the value of the expression $(-4)+(+2)$?

**7.** **a.** Model the expression $(+1)-(+2)$; then find the difference.

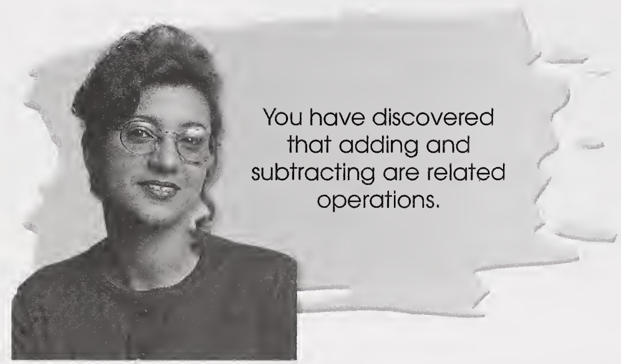
**b.** Model the expression $(+1)+(-2)$; then find the sum.

**c.** What do you notice about the value of the expression $(+1)-(+2)$ and the value of the expression $(+1)+(-2)$?

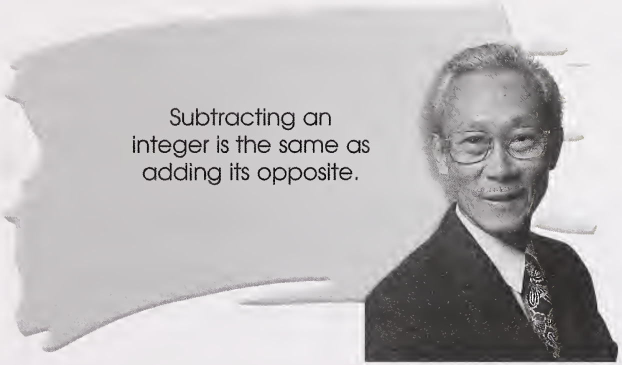
Check your answers by turning to the Appendix, page 128.

Look again at these statements:

$$(+1)-(+2)=-1$$
$$(+1)+(-2)=-1$$



This rule will help you subtract integers mentally.




Changing subtraction expressions to addition expressions usually makes them easier to solve.

**8.** Mentally compute each of the following differences; then write the answers. **Hint:** Rewrite each subtraction question as an addition question.

**a.** $(-7)-(-9)$ **b.** $(+4)-(+6)$ **c.** $(-3)-(+5)$

**d.** $\begin{array}{r}(+6)\\ \underline{-(-3)}\end{array}$ **e.** $\begin{array}{r}(-8)\\ \underline{-(-2)}\end{array}$ **f.** $\begin{array}{r}(+9)\\ \underline{-(+7)}\end{array}$

Check your answers by turning to the Appendix, page 131.

You may find a real-world application helpful in visualizing the subtraction of integers.

## Example

A chinook is a warm, dry wind that sometimes blows down the eastern side of the Rocky Mountains and across the prairies of Western Canada. Chinooks can raise the temperature by several degrees in a short time.

Because of a chinook, the temperature in Medicine Hat was $-8°C$ at 11:00 and 15°C at 13:00. What was the magnitude and direction of the temperature change?

## Solution

In order to solve this problem, you can write a mathematical expression involving the subtraction of integers.

temperature at 11:30

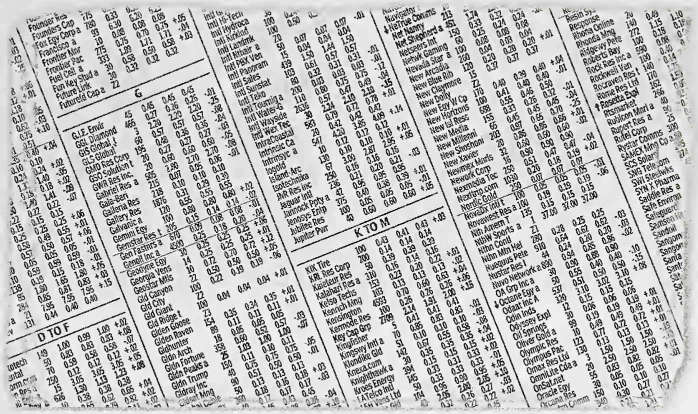
$$\underset{\text{temperature at 13:00}}{(+15)}-(-8)=(+15)+(+8)$$

$$=+23 \leftarrow \text{change in temperature}$$

The temperature increased 23°C.

9. Write a mathematical expression to describe each of the following situations. Then evaluate each expression.

   a. The opening share price of Allison Products was $5. The closing share price was $2. What was the direction and the magnitude of the price change?



   b. A hotel has underground parking, a lobby on the ground floor, and guest rooms on the floors above the lobby. The elevator travelled from the second parking level to the third floor above the lobby. What was the direction and magnitude of the change in position of the elevator?

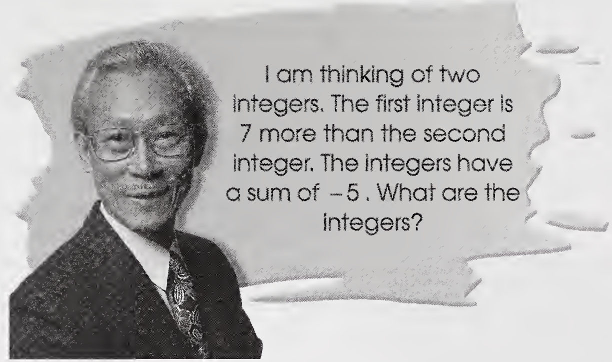
   c. A submarine was 3 km below sea level at 9:00. It was 4 km below sea level at 10:00. What was the direction and magnitude of the change in position of the submarine?

Check your answers by turning to the Appendix, page 131.

## Now Try This

Use a problem-solving strategy to answer the following questions.

10.




11. A ladder hangs over the side of the ship in the photograph. Right now, 5 m of the ladder is submerged. Later, if the tide rises 2 m, how much of the ladder is submerged?


Check your answers by turning to the Appendix, page 131.

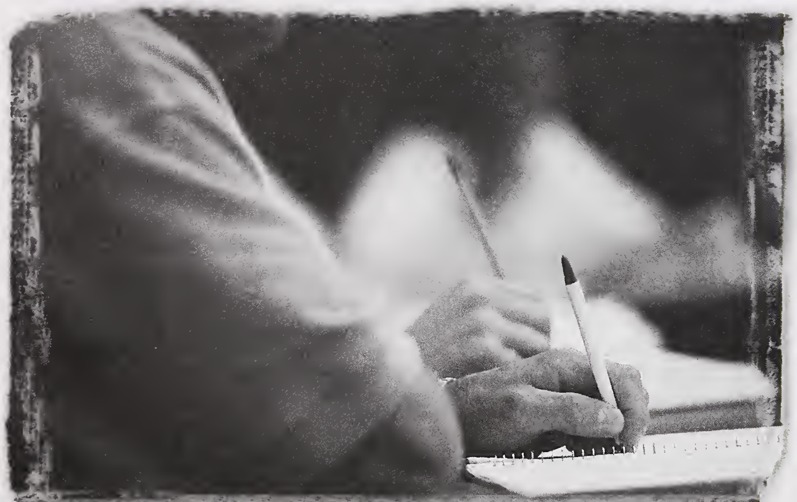
## Looking Back

In this activity, you modelled addition and subtraction of integers with concrete materials. You discovered patterns and applied the patterns to add or subtract integers mentally.

**12.** In your journal, write down an explanation of how to do the following.

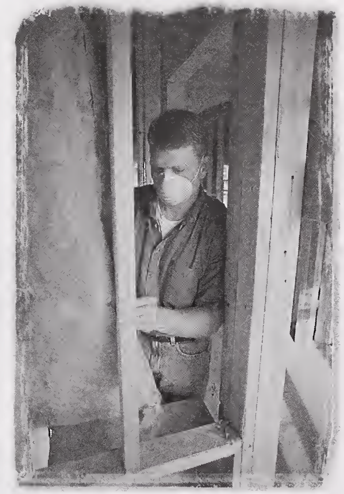
**a.** add integers with like signs
**b.** add integers with unlike signs
**c.** subtract integers

Check your answers by turning to the Appendix, page 131.



# Activity 3: Multiplying and Dividing Integers

Learning mathematics is like constructing a building. New skills are added to previously developed skills just as walls are built on a foundation.



In the previous activity, you reviewed how to add and subtract integers. In this activity, you will review how to multiply and divide integers.

## Multiplying of Integers

You can use counters to help you visualize the multiplication of integers. Remember $2 \times 3$ means "two groups of 3." Two groups of 3 can be arranged as an array with two rows and three columns.

Work through the following example.

### Example

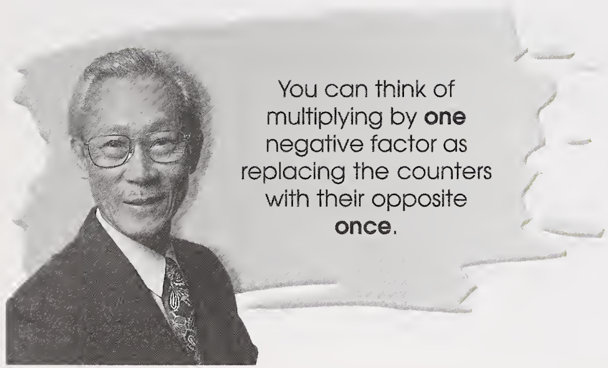
Use modelling to evaluate the expression $(+2)\times(+3)$. **Note:** This expression may also be written as $2 \times 3$.

## Solution

The expression $2 \times 3$ can be modelled with two rows of three positive counters.


$\therefore\ (+2)\times(+3)=+6$

What if one of the factors is a negative integer?

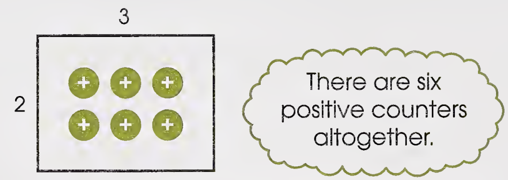



## Example

Use modelling to evaluate the expression $(-2)\times(+3)$. **Note:** This expression may also be written as $-2\times 3$.

## Solution

**Step 1:** First, model $(+2)\times(+3)$. The expression $2\times 3$ can be modelled with two rows of three positive counters.




**Step 2:** There is **one** negative factor in the expression $(-2)\times(+3)$. So, replace the six positive counters with their opposites **once**.


$\therefore\ (-2)\times(+3)=-6$

1. **a.** Model the expression $(+3)\times(+4)$; then find the product.

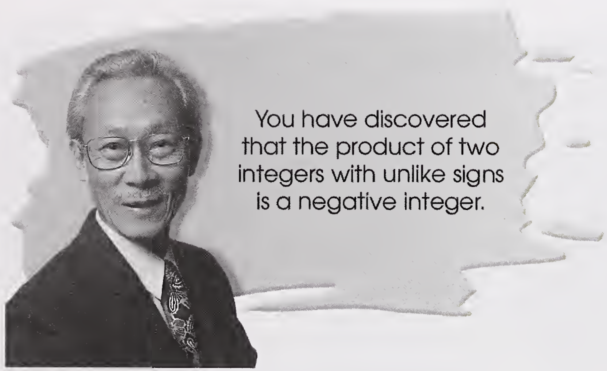
   **b.** Model the expression $(+5)\times(+5)$; then find the product.

2. **a.** Model the expression $(-3)\times(+4)$; then find the product.

   **b.** Model the expression $(+5)\times(-5)$; then find the product.

3. What generalization can you make about multiplying a positive integer and a negative integer?

Check your answers by turning to the Appendix, page 132.




For example, you have discovered that $(-3)\times(+4)=-12$ and $(+5)\times(-5)=-25$.

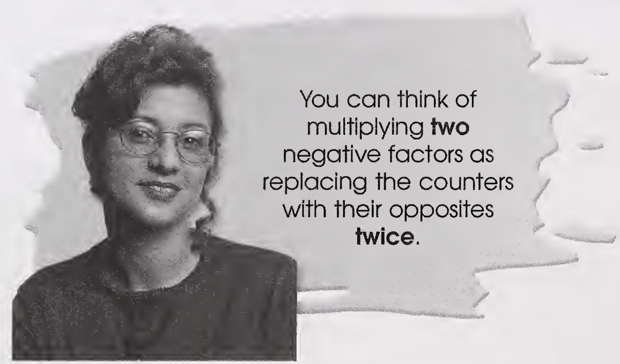
You can use patterns to confirm this generalization.

4. Use patterns to complete the following:

$$\begin{aligned}
3\times5&=15 \\
&\quad\text{subtract 5} \\
2\times5&=10 \\
&\quad\text{subtract 5} \\
1\times5&=5 \\
&\quad\text{subtract 5} \\
0\times5&=0 \\
&\quad\text{subtract 5} \\
-1\times5&=\square \\
&\quad\text{subtract 5} \\
-2\times5&=\square \\
&\quad\text{subtract 5} \\
-3\times5&=\square
\end{aligned}$$

Check your answers by turning to the Appendix, page 134.

What if you need to multiply **two** negative integers?

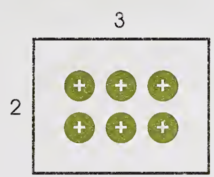


## Example

Use modelling to evaluate the expression $(-2)\times(-3)$. **Note:** This expression may also be written as $-2\times(-3)$.

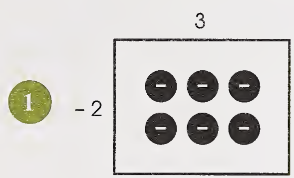
## Solution

**Step 1:** Model $2\times3$. The expression $2\times3$ can be modelled with two rows of three positive counters.

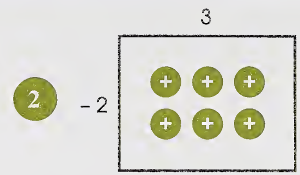


There are six positive counters altogether.

**Step 2:** There are **two** negative factors in the expression $(-2)\times(-3)$. So, replace the six positive counters with their opposites **twice**.



From replacing the counters once, there are six negative counters altogether.



From replacing the counters the second time, there are **six positive counters** altogether.

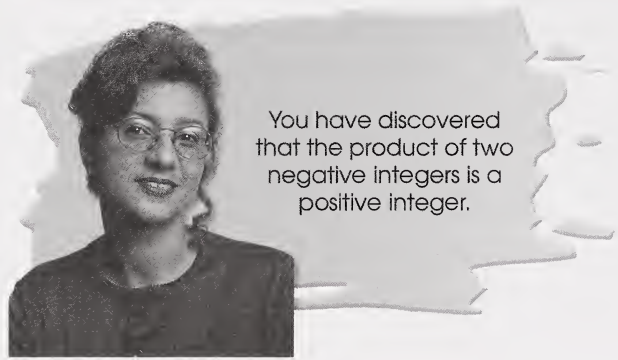
$\therefore\ (-2)\times(-3)=+6$

5. **a.** Model the expression $(-3)\times(-4)$; then find the product.

   **b.** Model the expression $(-5)\times(-5)$; then find the product.

6. What generalization can you make about multiplying two negative integers?

Check your answers by turning to the Appendix, page 134.



You can use patterns to confirm this generalization.

7. Use patterns to complete the following.

$$\begin{aligned} 3\times(-5) &= -15 \\ 2\times(-5) &= -10 \\ 1\times(-5) &= -5 \\ 0\times(-5) &= 0 \\ -1\times(-5) &= \\ -2\times(-5) &= \end{aligned}$$

add 5
add 5
add 5
add 5
add 5

Check your answers by turning to the Appendix, page 135.

You have discovered two rules you can use when multiplying integers.

- The product of two integers with like signs is a positive integer.

  (+) × (+) = (+)

- The product of two integers with unlike signs is a negative integer.

  (+) × (−) = (−)

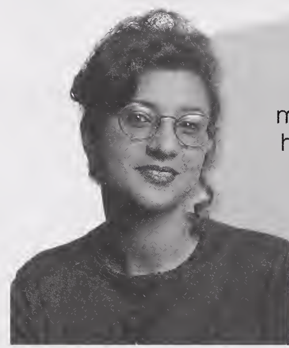
  (−) × (+) = (−)

8. Find each of the products.

a. $(-4)\times(+3)$ b. $(-4)\times(+4)$ c. $(-5)\times(-2)$

d. $\begin{array}{r}(+2)\\ \underline{\times(-6)}\end{array}$ e. $\begin{array}{r}(+3)\\ \underline{\times(+9)}\end{array}$ f. $\begin{array}{r}(-7)\\ \underline{\times(-3)}\end{array}$

Check your answers by turning to the Appendix, page 135.



When visualizing the multiplication of integers, it is helpful to have a real world context. Work through the following example.

## Example

Michelle likes to climb the cliffs of Nova Scotia's Bay of Fundy. At low tide, the cliffs are exposed, and the high water line can be seen on the cliffs.

If Michelle's present position is at the high water line, and she is moving up the cliff at a rate of 2 m per minute, what was her position 5 min ago?

## Solution

To solve this problem, you can write a mathematical expression involving the multiplication of integers.

$$\underbrace{(-5)}_{\text{number of minutes in the past}}\times\underbrace{(+2)}_{\text{distance travelled per minute}}=\underbrace{-10}_{\text{Michelle's position 5 min ago}}$$

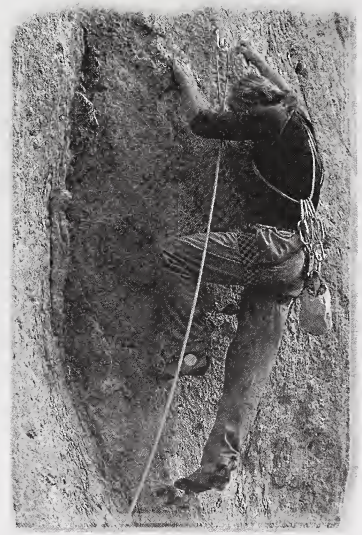
Michelle's position 5 min ago, was 10 m below the water line.

**9.** Write a mathematical expression to describe each of the following situations about Michelle and her rock climbing; then evaluate each expression.

**a.** If Michelle's present position is at the high water line and she is climbing up the cliff at a rate of 2 m per minute, what will be her position in 3 min?

**b.** If Michelle's present position is at the high water line and she is climbing down the cliff at a rate of 2 m per minute, what was her position 3 min ago?

**c.** If Michelle's present position is at the high water line and she is climbing down the cliff at a rate of 2 m per minute, what will be her position in 3 min?

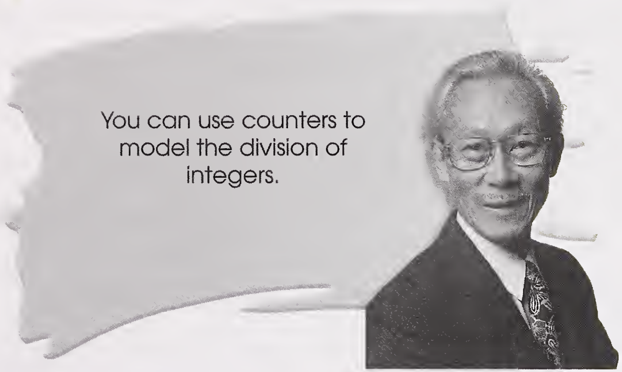


**10.** Megan is climbing up the face of a cliff. She is climbing at a rate of 2 m per minute.

**a.** How far above or below her present position was Megan 3 min ago?

**b.** How far above or below her present position will Megan be in 4 min?

Check your answers by turning to the Appendix, page 135.




## Dividing Integers

Work through the following example.

### Example

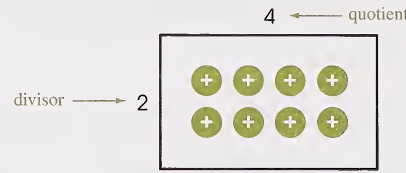
Use modelling to evaluate the expression $(-8) \div (+2)$. **Note:** This expression may also be written as $-8 \div 2$.

### Solution

**Step 1:** Model $8 \div 2$. Remember that the quotient in the statement $8 \div 2 = \square$ is the same as the missing factor in the statement $2 \times \square = 8$.

Ask yourself, "8 positive counters can be arranged in 2 rows of **how many**?"

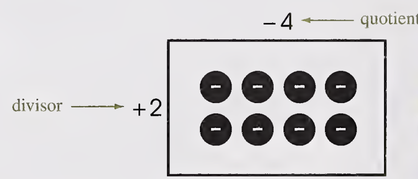
Arrange the 8 positive counters into the 2 rows.




The answer is, "8 positive counters can be arranged in 2 rows of **4**."

**Step 2:** Use the model in Step 1 to model $(-8) \div (+2)$.

The quotient will be either $+4$ or $-4$. Because the dividend is $-8$, the counters must have been exchanged for their opposites once. Because the divisor is $+2$, the quotient must be $-4$. (One negative factor is required to exchange counters for their opposites.)




$\therefore \ (-8) \div (+2) = -4$

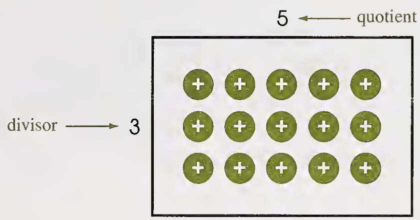
## Example

Use modelling to evaluate the expression $(+15)\div(-3)$. **Note:** This expression may also be written as $15\div(-3)$.

## Solution

**Step 1:** Model $15\div 3$. Remember that the quotient in $15\div 3=\square$ is the same as the missing factor in $3\times\square=15$.

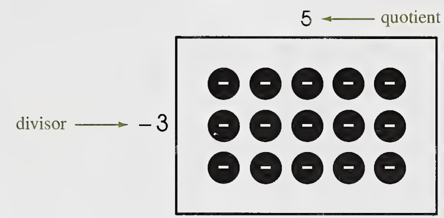
Ask yourself, "15 positive counters can be arranged in 3 rows of **how many**?"

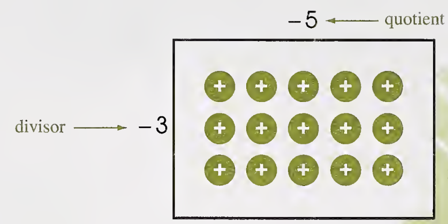



The answer is, "15 positive counters can be arranged in 3 rows of **5**."

**Step 2:** Use the model in Step 1 to model $(+15)\div(-3)$.

The quotient will be either $+5$ or $-5$. Because the dividend $(+15)$ is positive and the divisor $(-3)$ is negative, the counters must have been exchanged for their opposites **twice**. (Two negatives are required to exchange counters for their opposites twice.)




$\therefore\ (+15)\div(-3)=-5$

11. Use counters to model each of the following mathematical expressions.

a. $(-12) \div (+4)$ b. $(+18) \div (-2)$

12. a. What do you notice about the sign of the quotient when you divide a positive integer and a negative integer?

b. What do you notice about the sign of the quotient when you divide two negative integers?

Check your answers by turning to the Appendix, page 136.

You can use the relationship between multiplication and division to write two rules that will help you divide integers mentally.

- The quotient of two integers with like signs is a positive integer.
- The quotient of two integers with unlike signs is a negative integer.

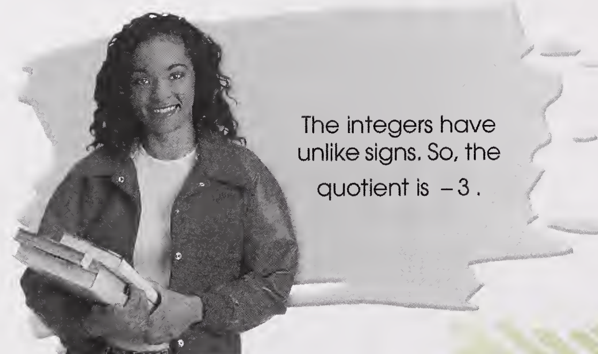
Work through the following examples.

### Example

Divide $-15$ by $+5$.

### Solution

$(+5)\overline{)(-15)}$



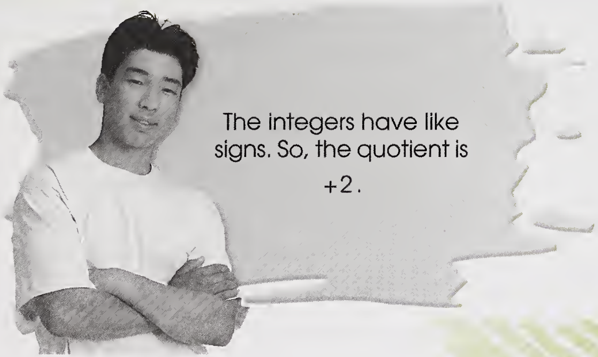

$\therefore\ (-15) \div (+5) = -3$

## Example

How many times does $-5$ go into $-10$?

## Solution

$(-10) \div (-5)$




$\therefore \; (-10) \div (-5) = +2$

**13.** Find the quotient for each of the following.

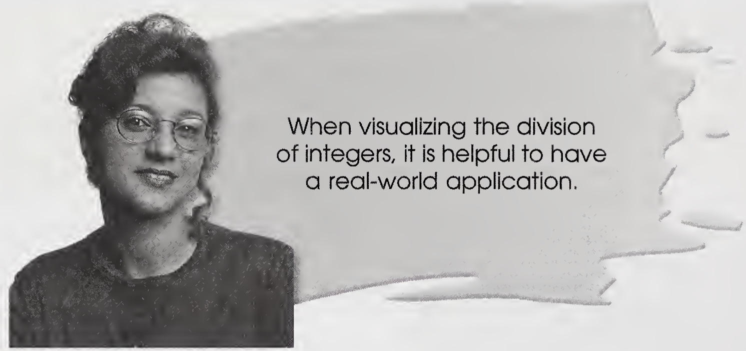
**a.** $(-8) \div (-2)$

**b.** $(-18) \div (+2)$

**c.** $(-3)\overline{)(-15)}$

**d.** $(+5)\overline{)(-10)}$

Check your answers by turning to the Appendix, page 137.




Work through the following example.

## Example

The present temperature is 0°C. If the temperature is falling 2°C each hour, how long will it take to reach $-10°C$?

## Solution

To solve this problem, you can write a mathematical expression involving division of integers.

$$\underbrace{(-10)}_{\text{change in temperature}} \div \underbrace{(-2)}_{\text{decrease in temperature per hour}} = \underbrace{5}_{\text{total number of hours}}$$

It will take 5 h.

14. Write a mathematical expression to describe each of the following situations. Then evaluate each expression.

a. A submarine is descending at the rate of 30 m per minute. How long will it take to descend 1800 m?

b. On Tuesday, the temperature dropped 10°C in 2 h. What was the average change in temperature per hour?

c. A chinook caused the temperature to rise 10°C in 5 min. What was the average change in temperature per minute?

d. When Fred turned his new freezer on, it took 6 h for the temperature to fall 30°C. What was the average change in temperature per hour?

e. A submarine descends at the rate of 100 m per minute. How many minutes will it take to descend from sea level to 2000 m below sea level?

Check your answers by turning to the Appendix, page 137.

## Looking Back

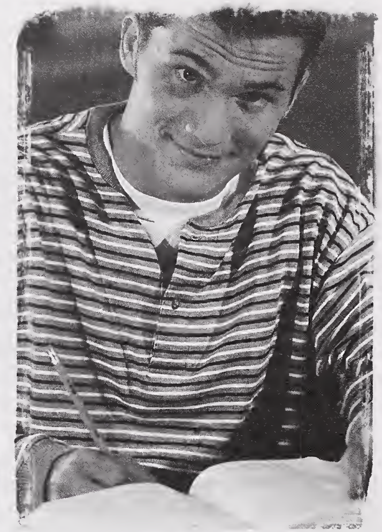
In this activity, you modelled multiplication and division of integers with concrete materials. You saw that multiplying by $-1$ means the counters are replaced with their opposites. Most people find multiplication and division of integers easier than addition and subtraction of integers.

15. In your journal, complete the following sentences using the word *positive* or *negative*.

a. The product of two positive numbers is ________.

b. The product of two negative numbers is ________.

c. The product of a positive and a negative number is ________.



Check your answers by turning to the Appendix, page 138.

## Conclusion

EDMONTON OILERS HOCKEY CLUB

In this section, you modelled the operations of addition, subtraction, multiplication, and division of integers with concrete materials.

You can see how the addition and subtraction of integers is used in the plus-minus system of the National Hockey League. At one point during the regular season, two players from the Edmonton Oilers, Janne Niinimaa and Bill Guerin, had 14 pts from their goals and assists. Yet, in the plus-minus system, Janne Niinimaa was a $-2$ while Bill Guerin was a $+16$. Thus, for the total time Janne Niinimaa was on the ice, the Oilers scored two fewer goals against their opponents than they had scored on them.

On the other hand, while Bill Guerin was on the ice, the Oilers scored 16 more goals than were scored against them. You can see that the plus-minus gives a very different picture of a player than the points a player accumulates.

To calculate plus-minus figures, you must be able to perform operations on integers.

## Assignment

Turn to Assignment Booklet 1B and complete the assignment for Section 2.

# SECTION 3

## Exponential Notation

Jennifer is doing a project on a computer. The computer monitor she is using produces colours by mixing the three primary screen colours—red, green, and blue. If the monitor is capable of producing 256 tones of each primary colour, how many colours can the monitor produce?

The computer produces $256^3$ colours. The exponent, 3, indicates that the base, 256, is multiplied by itself 3 times.

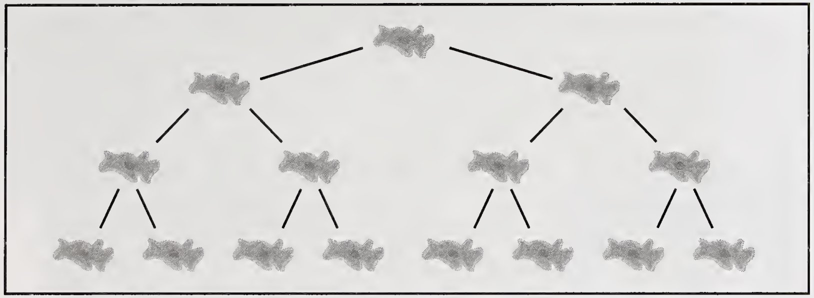
$$256^3 = 256 \times 256 \times 256 \text{ or } 16\,777\,216$$

Therefore, Jennifer's computer monitor is capable of producing 16 777 216 colours.

Throughout this section, you will work with exponential notation. You will review and increase your knowledge of powers, develop rules for calculating powers by recognizing patterns, and calculate solutions to questions and problems involving powers.

## Activity 1: Exploring Powers

An amoeba is a single-celled animal that reproduces by a process called cell division. One cell divides into two cells; then each of these cells divides into two more cells. In a few hours, a single amoeba can become a colony of amoebas.



To visualize the increase in amoebas, make a model by performing the following steps:

**Step 1:** Obtain a large rectangular sheet of paper. (You may use a newspaper page.)

**Step 2:** Fold the sheet in half. The fold represents a division of an amoeba. Each sheet resulting from the fold represents an amoeba.

**Step 3:** Continue to fold the paper in half until several layers are formed.

**1.** Copy and complete the following table by counting the number of layers of paper after each fold.

| Number of Folds | Number of Layers |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | |
| 4 | |

**2.** Amoeba cell division follows the same pattern as the folding paper example. Use the folding paper example to predict the number of amoebas there are after the following.

**a.** no divisions **b.** one division **c.** two divisions

**d.** three divisions **e.** four divisions **f.** five divisions

Check your answers by turning to the Appendix, page 138.

So far in this activity, you have used a number in standard form to represent the number of amoebas after each split.

**The standard form of a number is the usual form of a number.**

Powers can also be used to describe the number of amoebas after each division.

**A power is an expression written with a base and an exponent. The base of a power shows the factor that is repeatedly multiplied. The exponent of a power shows the number of times the base is used as a factor. For example, $3^5$ is a power where the base is 3 and the exponent is 5. This represents $3 \times 3 \times 3 \times 3 \times 3$.**

3. The following table shows how to express the number of amoebas in three different ways: in standard form, factored form, or power form.

| Number of Divisions | Number of Amoebas at Each Division | | |
|---|---|---|---|
| | Standard Form | Factored Form | Power Form |
| 1 | 2 | 2 | $2^1$ |
| 2 | 4 | $2 \times 2$ | $2^2$ |
| 3 | 8 | $2 \times 2 \times 2$ | |
| 4 | 16 | $2 \times 2 \times 2 \times 2$ | |
| 5 | 32 | $2 \times 2 \times 2 \times 2 \times 2$ | |

a. Complete the table.

b. How many amoebas will there be after 6 divisions? Express the answer in standard form.

c. How many amoebas will there be after 6 divisions? Express the answer in power form.

d. What is the factored form of $2^6$?

e. What is the relationship between the number of factors in the factored form and the exponent in the power form?

Check your answers by turning to the Appendix, page 138.

Powers can be read in different ways:

- The expression $2^2$ is read as "two squared."
- The expression $2^3$ is read as "two cubed."
- The expression $2^4$ is read as "two to the fourth."
- The expression $2^5$ is read as "two to the fifth."

## Evaluating Powers

To evaluate a power using paper and pencil, write the power as its base multiplied out as many times as it has factors.

Work through the following example.

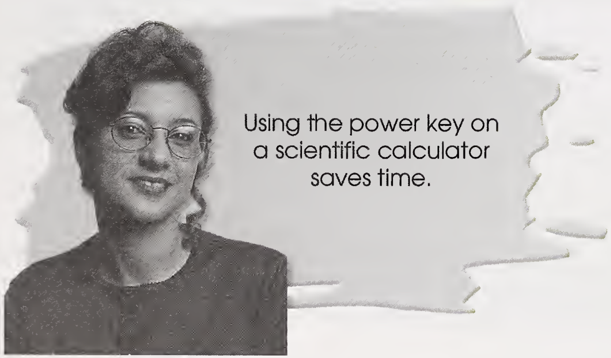
### Example

Evaluate $5^4$.

### Solution

$$5^4 = 5\times5\times5\times5 \quad \textbf{or} \quad 5^4 = 5\times5\times5\times5$$
$$= 25\times5\times5 \qquad\qquad = (5\times5)\times(5\times5)$$
$$= 125\times5 \qquad\qquad = 25\times25$$
$$= 625 \qquad\qquad = 625$$

It is sometimes helpful to group factors rather than work from left to right.




Work through the following example using a scientific calculator.

### Example

Evaluate $8^4$.

### Solution

8 $x^y$ 4 =

4096.

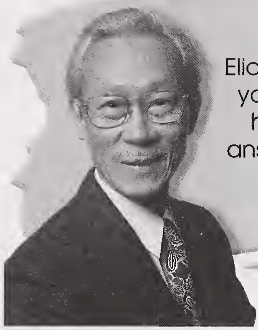
**Note:** All scientific calculators and graphing calculators have a power key, but the labels for this key vary. Sometimes the power key will look like $x^y$, $y^x$, or ^.

**4.** Evaluate each of the following using paper and pencil. Then confirm your answers using the power key on your calculator.

**a.** $5^6$ **b.** $13^3$ **c.** $39^4$ **d.** $1.7^3$

Check your answers by turning to the Appendix, page 138.

When using paper and pencil to evaluate a power with a negative base, write out the factors. You can then count the number of signs and decide if the answer will be positive or negative.





Work through the following example.

### Example

Evaluate $(-5)^4$.

### Solution

$$\begin{aligned}(-5)^4 &= (-5)\times(-5)\times(-5)\times(-5)\\ &= +(5\times5\times5\times5)\\ &= +625\end{aligned}$$

There are four negative signs; thus, the answer is positive.

### Example

Evaluate $(-0.4)^3$.

### Solution

$$\begin{aligned}(-0.4)^3 &= (-0.4)\times(-0.4)\times(-0.4)\\ &= -(0.4\times0.4\times0.4)\\ &= -0.064\end{aligned}$$

There is an odd number of negative signs; thus, the answer is negative.

**5.** Evaluate the following negative bases using paper and pencil.

**a.** $(-6)^3$ **b.** $(-0.5)^2$ **c.** $(-3)^5$

Check your answers by turning to the Appendix, page 138.

## Powers and Their Opposites

Off is the opposite of on. Turning a screw to the left is the opposite of turning it to the right. Negative is the opposite of positive.

You know that numbers like $-12$ and 12 are opposites. Powers have opposites too, but the opposite may not always be obvious.

### Example

What is the opposite of $2^3$? Explain.

### Solution

$$\begin{aligned} 2^3 &= 2\times2\times2 \\ &= 8 \end{aligned} \qquad \begin{aligned} -2^3 &= -(2\times2\times2) \\ &= -8 \end{aligned}$$

Therefore, the opposite of $2^3$ is $-2^3$.

**Note:** The expression $-2^3$ is read as "the opposite of two cubed."

### Example

What is the opposite of $(-5)^4$? Explain.

### Solution

$$\begin{aligned} (-5)^4 &= (-5)\times(-5)\times(-5)\times(-5) \\ &= 625 \end{aligned}$$

$$\begin{aligned} -(-5)^4 &= -\left[(-5)\times(-5)\times(-5)\times(-5)\right] \\ &= -625 \end{aligned}$$

Therefore, the opposite of $(-5)^4$ is $-(-5)^4$.

Also, $\begin{aligned} -5^4 &= -(5\times5\times5\times5) \\ &= -625 \end{aligned}$

The opposite of $(-5)^4$ is also $-5^4$.

**Note:** The expression, $(-5)^4$, is read as "negative five to the fourth." The expression, $-5^4$, is read as "the opposite of five to the fourth."

The presence or absence of parentheses is very important when you are working with positive and negative signs. The parentheses in $(-5)^4$ are used to indicate that the negative sign is part of the base.

6. Indicate the sign of the standard form of each power. Do not actually evaluate each power.

a. $-8^2$  b. $(-8)^3$  c. $(-8)^4$

d. $-(-3)^2$  e. $-(-2)^5$

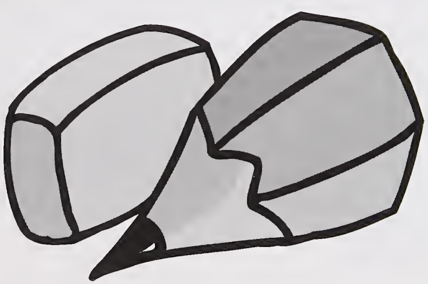
7. Indicate which is greater in each of the following pairs of powers.

a. $(-4)^2$ or $-4^2$  b. $-3^4$ or $-(-3)^4$

8. Evaluate each of the following expressions.

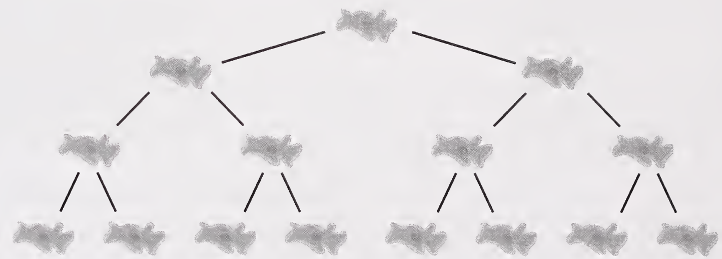
a. $-6^8$  b. $(-6)^8$  c. $-(-5)^7$  d. $-5^7$

Check your answers by turning to the Appendix, page 139.



## Powers with Exponents of Zero

Think about the problem involving the cell division of an amoeba at the beginning of this activity.



9. a. How many amoebas are there when no divisions have occurred (zero divisions)?

b. What power can be used to describe the number of amoebas after zero divisions?

10. a. What does $5^0$ mean? To find out, complete the following table.

| Power Form | Standard Form |
|---|---|
| $5^4$ | 615 |
| $5^3$ | 125 |
| $5^2$ | 25 |
| $5^1$ | 5 |
| $5^0$ | |

Pattern

Divide by 5.

Divide by 5.

Divide by 5.

Divide by 5.

**b.** Confirm the value of $5^0$ by using the power key on your scientific calculator. That is, press the following key sequence.

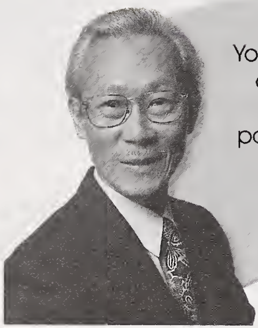
$\boxed{5}\ \boxed{x^y}\ \boxed{0}\ \boxed{=}$

**c.** What is the value of $(-3)^0$?

**d.** What can you conclude about powers with an exponent of zero?

Check your answers by turning to the Appendix, page 139.

## Powers with Negative Exponents



You used patterns to discover that a power with a zero exponent equals 1. You can also use patterns to discover the meaning of a power with a negative exponent.

**11.** What does $2^{-4}$ mean? What is its value? To find out, complete the following table.

| Power Form | Standard Form | Pattern |
|---|---|---|
| $2^2$ | 4 | Divide by 2. |
| $2^1$ | 2 | Divide by 2. |
| $2^0$ | 1 | Divide by 2. |
| $2^{-1}$ | $\frac{1}{2}$ | Divide by 2. |
| $2^{-2}$ | $\frac{1}{4}$ | Divide by 2. |
| $2^{-3}$ | $\frac{1}{8}$ | Divide by 2. |
| $2^{-4}$ | | Divide by 2. |
| $2^{-5}$ | | Divide by 2. |
| $2^{-6}$ | | |

**12.** Examine the table in question 11. Then explain how each of the following pairs of powers are related.

**a.** $2^2$ and $2^{-2}$

**b.** $2^1$ and $2^{-1}$

**13.** What is the relationship between the exponents of each of the pairs of powers in question 12?

Check your answers by turning to the Appendix, page 140.

After completing questions 11 to 13, you should be able to see that a negative exponent indicates that a reciprocal is to be used.

Work through the following example.

## Example

Express each of the following powers with a positive exponent.

**a.** $5^{-4}$  **b.** $1.3^{-2}$  **c.** $(-5)^{-3}$

## Solution

**a.** The power $5^{-4}$ means "take the reciprocal of $5^4$." The reciprocal of $5^4$ is $\frac{1}{5^4}$.

$\therefore\ 5^{-4} = \frac{1}{5^4}$

**b.** The power $1.3^{-2}$ means "take the reciprocal of $1.3^2$." The reciprocal of $1.3^2$ is $\frac{1}{1.3^2}$.

$\therefore\ 1.3^{-2} = \frac{1}{1.3^2}$

**c.** The power $(-5)^{-3}$ means "take the reciprocal of $(-5)^3$." The reciprocal of $(-5)^3$ is $\frac{1}{(-5)^3}$.

$\therefore\ (-5)^{-3} = \frac{1}{(-5)^3}$

**14.** Express each of the following with a positive exponent.

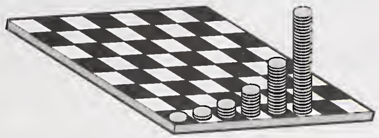
**a.** $4^{-3}$  **b.** $(-7)^{-4}$  **c.** $0.6^{-5}$

Check your answers by turning to the Appendix, page 141.

## Now Try This

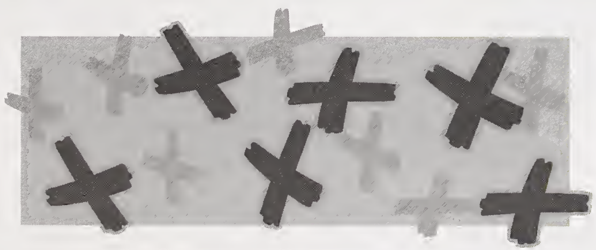
Use a problem-solving strategy to answer questions 15 and 16.

**15.** Suppose you were to stack loonies on a chessboard in a way such that you have 1 loonie on the first square, 2 loonies on the second square, 4 loonies on the third square, 8 loonies on the fourth square, and so on. What square will be the first to have over $1 000 000 on it? Explain how to arrive at this answer.



**16.** Twice a number is always smaller than the number squared. What is the smallest whole number for which this statement is always true? Show how you found this number.

Check your answers by turning to the Appendix, page 141.



## Looking Back

In this activity, you saw that exponential notation is used in cases where repeated multiplication occurs. You already knew that multiplication is used for repeated addition. You discovered some basic rules about powers, ways to evaluate powers, and explored several patterns in powers.

**17.** You know $2 \times 500 = 1000$. If you were adding 2 to itself, you would have to add five hundred times before you would reach a 4-digit number.

$$\underbrace{2+2+2+\ldots+2}_{500\text{ times}} = 1000$$

In your journal, copy the following sentences and fill in the blanks with the appropriate number.

If you were **multiplying** 2 by itself, a 4-digit number would result much quicker! In fact, the minimum number of times 2 has to be multiplied by itself to obtain a 4-digit number is ________ and the maximum number of times is ________.

**Hint:** Use the $\boxed{x^y}$ key of your calculator to help you fill the blanks.

Check your answer by turning to the Appendix, page 141.

# Activity 2: Multiplying and Dividing Powers

A very simple pattern is repeated many times to make the design on the right. Often, in your work in mathematics, you look for patterns you can apply in order to perform computations. In this activity, you will look for patterns in multiplying and dividing powers.

## Multiplying Powers

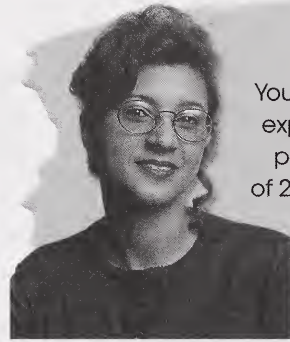
In the section introduction, you were told that a colour computer monitor can produce over 16 million colours using the three primary colours of light: red, blue, and green. You also found that the number of colours produced can be written as $256 \times 256 \times 256$ or $256^3$.

Using a scientific calculator, you can determine exactly how many colours can be produced.

2 5 6 $x^y$ 3 =

16777216.

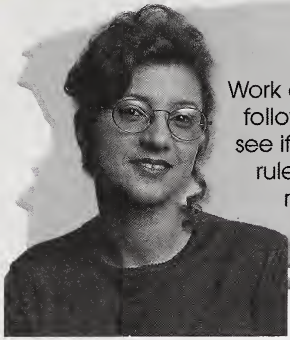
The colour monitor can display 16 777 216 different colours.



You can also write the expression $256^3$ as a power with a base of 2 because $256 = 2^8$.

$$\begin{aligned} 256^3 &= 256 \times 256 \times 256 \\ &= 2^8 \times 2^8 \times 2^8 \\ &= (2\times2\times2\times2\times2\times2\times2\times2)\times(2\times2\times2\times2\times2\times2\times2\times2) \\ &\quad \times(2\times2\times2\times2\times2\times2\times2\times2) \\ &= 2^{24} \end{aligned}$$

Count the number of 2s.



Work carefully through the following questions and see if you can discover a rule that will help you multiply powers.

1. Complete the following table. The first row is done as an example.

| Expression | Factored Form | Power Form |
|---|---|---|
| $3^4 \times 3^3$ | $(3\times3\times3\times3)\times(3\times3\times3)$ $=3\times3\times3\times3\times3\times3\times3$ | $3^7$ |
| $4^2 \times 4^3$ | | |
| $5 \times 5^3$ | | |
| $2^2 \times 2^3 \times 2^4$ | | |

2. What shortcut method do you see for multiplying powers?

3. Explain why the shortcut does not work for $3^2 \times 4^5$.

Check your answers by turning to the Appendix, page 141.

You have just discovered a rule for multiplying powers with the same base.

**To multiply powers that have the same base, add the exponents and use the same base.**

## Example

Write each of the following as a single power.

**a.** $3^4 \times 3^5$  **b.** $(-4)^3 \times (-4)^5$

**c.** $3^{-2} \times 3^{-1}$

## Solution

**a.** $3^4 \times 3^5 = 3^{4+5}$
$= 3^9$

**b.** $(-4)^3 \times (-4)^5 = (-4)^{3+5}$
$= (-4)^8$

**c.** $3^{-2} \times 3^{-1} = 3^{-2+(-1)}$
$= 3^{-3}$

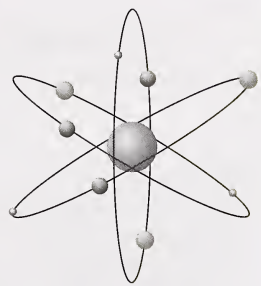
4. Where possible, write each of the following as a single power.

**a.** $2^3 \times 2^7$  **b.** $(-5)^8 \times (-5)^{10}$  **c.** $0.5^3 \times 2.5^4$

**d.** $8^3 \times 8^4 \times 8^5$  **e.** $8^2 \times 8^{-3}$  **f.** $2^{-3} \times 2^{-2}$

5. In an atom, an electron makes about $10^{15}$ orbits per second around the nucleus. How many orbits does it make in the following times.

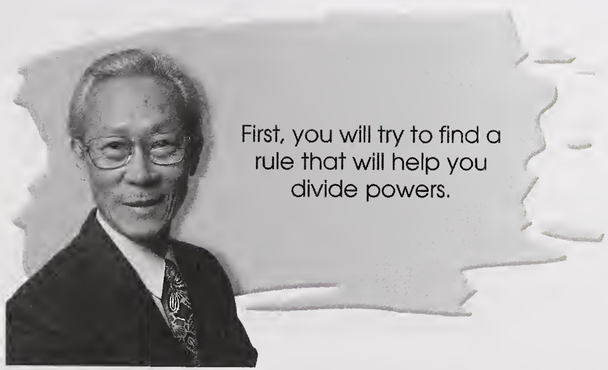
**a.** $10^4$ s  **b.** $10^{11}$ s



6. Dale has a stereo system with an amplifier as well as a preamplifier. The preamplifier boosts the signal by a factor of $10^5$, and the amplifier boosts the signal by a factor of $10^4$. Calculate the factor by which the signal is boosted after it passes through both amplifiers.

Check your answers by turning to the Appendix, page 142.

## Dividing Powers




7. Complete the following table. The first two are done as examples.

| Expression | Factored Form | Power Form |
|---|---|---|
| $5^6 \div 5^2$ | $(5\times5\times5\times5\times5\times5)\div(5\times5)$ <br> $=\dfrac{\overset{1}{\cancel{5}}\times\overset{1}{\cancel{5}}\times5\times5\times5\times5}{\underset{1}{\cancel{5}}\times\underset{1}{\cancel{5}}}$ <br> $=5\times5\times5\times5$ | $5^4$ |
| $2^5 \div 2^3$ | $(2\times2\times2\times2\times2)\div(2\times2\times2)$ <br> $=\dfrac{\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times\overset{1}{\cancel{2}}\times2\times2}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{2}}}$ <br> $=2\times2$ | $2^2$ |
| $4^6 \div 4^5$ | | |
| $3^7 \div 3^4$ | | |
| $5^5 \div 5^1$ | | |

8. What shortcut method do you see for dividing powers?

9. Explain why the rule does not work for $5^6 \div 2^3$.

Check your answers by turning to the Appendix, page 142.

You have just discovered a rule for dividing powers with the same base.

**To divide powers that have the same base, subtract the exponents and use the same base.**

## Example

Write each of the following as a single power.

a. $\dfrac{8^5}{8^2}$  b. $(-3)^4 \div (-3)^2$  c. $5^{-4} \div 5^{-2}$

### Solution

a. $\dfrac{8^5}{8^2} = 8^{5-2}$
$= 8^3$

b. $(-3)^4 \div (-3)^2 = (-3)^{4-2}$
$= (-3)^2$

c. $5^{-4} \div 5^{-2} = 5^{-4-(-2)}$
$= 5^{-4+2}$
$= 5^{-2}$

**10.** Where possible, write each of the following as a single power.

a. $3^6 \div 3^2$  b. $(-5)^7 \div (-5)^3$  c. $\dfrac{5^7}{5^7}$

d. $\dfrac{7^{12}}{12^7}$  e. $(-3)^2 \div (-3)^5$  f. $4^{-2} \div 4^{-4}$

Check your answers by turning to the Appendix, page 143.

## Now Try This

**11.** Write each of the following as a single power.

a. $\dfrac{8^2 \times 8^3}{8^4}$  b. $\dfrac{9^2 \times 9^3 \times 9^4}{9 \times 9^5}$

c. $(4^2 \times 4^6 \times 4) \div (4^5 \times 4 \times 4)$

Check your answers by turning to the Appendix, page 143.

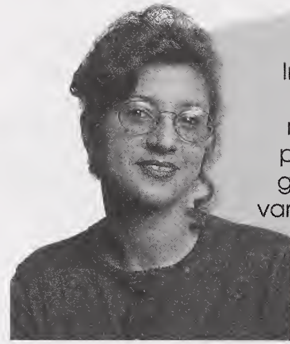
## Non-Numeric Bases

In Activity 1, you used powers to represent the number of amoebas after each division.

Since the exponent in the power used to represent the number of amoebas is the number of divisions, you can write a general form for the number of amoebas as $2^n$.

| Number of Divisions | Number of Amoebas |
|---|---|
| 1 | $2^1$ |
| 2 | $2^2$ |
| 3 | $2^3$ |
| 4 | $2^4$ |
| 5 | $2^5$ |
| • | • |
| • | • |
| • | • |
| $n$ | $2^n$ |

The amoeba example shows that $2^n$ means 2 multiplied by itself $n$ times.



In this activity, you discovered some properties or rules for multiplication and division of powers. Now you are going to generalize those rules by using variables to replace the numbers.

When multiplying powers with the same base, add the exponents.

$$n^a \times n^b = n^{a+b}$$

### Example

Simplify $n^3 \times n^2$.

### Solution

$$\begin{aligned} n^3 \times n^2 &= n^{3+2} \\ &= n^5 \end{aligned}$$

**Note:** $n^3 \times n^2$ means $(n \times n \times n) \times (n \times n)$.

### Example

Simplify $n^a \times n^b$.

### Solution

$n^a \times n^b = n^{a+b}$

**Note:** $n^a \times n^b$ means $\underbrace{(n \times n \times n \times \ldots \times n)}_{a \text{ factors}} \times \underbrace{(n \times n \times n \times \ldots \times n)}_{b \text{ factors}}$.

Similarly, to divide powers with the same base, subtract the exponents. Recall that division by zero is undefined; therefore, the base cannot be zero.

$$n^a \div n^b = n^{a-b}$$

### Example

Simplify $n^7 \div n^2$.

### Solution

$n^7 \div n^2 = n^{7-2}$
$\quad = n^5$

**Note:** $n^7 \div n^2 = \dfrac{n \times n \times n \times n \times n \times n \times n}{n \times n}$

### Example

Simplify $n^a \div n^b$.

### Solution

$n^a \div n^b = n^{a-b}$

**Note:** $n^a \div n^b = \underbrace{(n \times n \times n \times \ldots \times n)}_{a \text{ factors}} \div \underbrace{(n \times n \times n \times \ldots \times n)}_{b \text{ factors}}$

**12.** Simplify the following.

**a.** $p \times p \times p \times p \times p \times p \times p$ **b.** $m^2 \times m^4$

**c.** $n \times n^2 \times n^3$ **d.** $p^8 \div p^3$

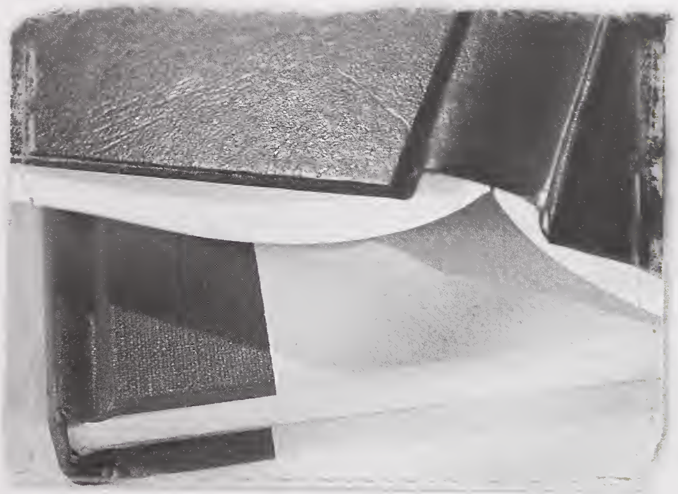
**e.** $\dfrac{a^6}{a^3}$ **f.** $\dfrac{a^3 b^6}{ab^2}$

Check your answers by turning to the Appendix, page 144.

## Looking Back

In this activity, you multiplied and divided numbers written in exponential form using both numeric and non-numeric bases.

**13.** In your journal, write down the rule for multiplying two powers that have the **same base** and the rule for dividing two powers that have the **same base**. Include examples to illustrate the rules.

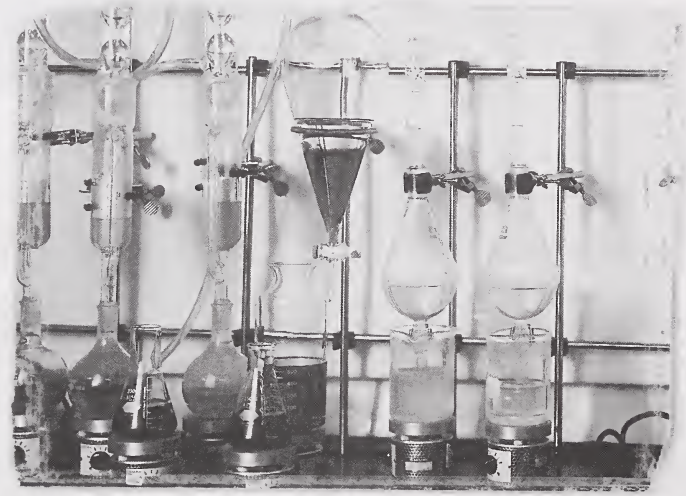


**14.** Explain why you cannot write the product $3^5 \times 5^3$ as a single power.

Check your answers by turning to the Appendix, page 144.

## Activity 3: Power Rules

A biochemistry laboratory carries out about $10^3$ experiments each year. If each experiment uses $10^{-2}$ g of calcium citrate, how many grams of calcium citrate are used each year in this lab?



Power rules allow you to solve problems like this by working directly with the exponents.

There are several rules that involve more than one power. These rules have extensive application in the area of science. In this activity, you will discover the rules for the power of a power, the power of a product, and the power of a quotient. You will then extend these rules to variable bases.

## Power of a Power

It has been estimated that each galaxy contains $10^{11}$ stars. If there are about $10^{11}$ galaxies in the universe, approximately how many stars are there in total? You can express the number of stars as $10^{11} \times 10^{11}$ or $\left(10^{11}\right)^2$.

What is $\left(10^{11}\right)^2$ equal to as a single power? As you work through the following question, see if you can discover a rule explaining how to simplify a power of a power.

1. Complete the following table. The first one is done as an example.

| Expression | Factored Form | Power Form |
|---|---|---|
| $\left(5^3\right)^2$ | $5^3 \times 5^3 = (5\times5\times5)\times(5\times5\times5)$ <br> $= 5\times5\times5\times5\times5\times5$ | $5^6$ |
| $\left(7^2\right)^3$ | | |
| $\left(3^4\right)^2$ | | |
| $\left(2^3\right)^4$ | | |

2. What shortcut method do you see for simplifying the power of a power?

Check your answers by turning to the Appendix, page 144.

**To simplify a power of a power, multiply the exponents and keep the same base.**

## Example

Express each of the following in simplest power form.

**a.** $\left(10^{2}\right)^{3}$

**b.** $\left[(-5)^{4}\right]^{2}$

**c.** $\left[(-2)^{2}\right]^{-3}$

### Solution

**a.** $\left(10^{2}\right)^{3} = 10^{2\times 3}$
$= 10^{6}$

**b.** $\left[(-5)^{4}\right]^{2} = (-5)^{4\times 2}$
$= (-5)^{8}$

**c.** $\left[(-2)^{2}\right]^{-3} = (-2)^{2\times(-3)}$
$= (-2)^{-6}$

**3.** Write each of the following as a single power.

**a.** $\left(3^{5}\right)^{4}$ **b.** $\left[(-2)^{3}\right]^{7}$ **c.** $\left(0.2^{5}\right)^{3}$

**d.** $\left[(-0.1)^{2}\right]^{6}$ **e.** $\left(4^{2}\right)^{-1}$ **f.** $\left(10^{-2}\right)^{-3}$

**4.** You may recall from previous mathematics courses that the terms googol and googolplex are used for very large numbers.

**a.** A googol is written as $\left(10^{10}\right)^{10}$. Write this number as a power with a single exponent.

**b.** A googolplex is $10^{\text{googol}}$ or $10^{\left(10^{10}\right)^{10}}$. Describe, in words, what the exponent would be for a googolplex written as a power with a single exponent.

**5.** Explain why $\left(3^{4}\right)^{2} = \left(3^{2}\right)^{4}$ by writing each as a product of equal factors.

Check your answer by turning to the Appendix, page 145.

## Power of a Product

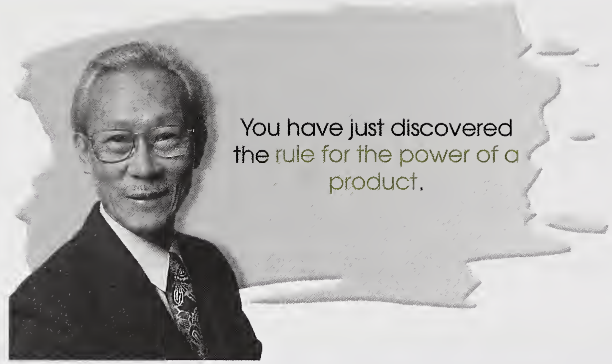
The expression $\left(8^2 \times 2^3\right)^5$ is called a power of a product. Is there a shorter way of writing $\left(8^2 \times 2^3\right)^5$? Your answers to the following question should help you discover a rule for simplifying a power of a product.

6. Complete the following table. The first one is done as an example.

| Expression | Factored Form | Power Form |
|---|---|---|
| $\left(6^2 \times 4^3\right)^3$ | $\left(6^2 \times 4^3\right) \times \left(6^2 \times 4^3\right) \times \left(6^2 \times 4^3\right)$ $= 6^2 \times 6^2 \times 6^2 \times 4^3 \times 4^3 \times 4^3$ | $6^6 \times 4^9$ |
| $\left(5^4 \times 2^7\right)^2$ | | |
| $\left(8 \times 7^4\right)^3$ | | |
| $\left(3^5 \times 4^6\right)^2$ | | |

7. What shortcut method do you see for simplifying the power of a product?

Check your answers by turning to the Appendix, page 145.




**To simplify a power of a product, multiply each of the exponents inside the brackets with the exponent outside the brackets.**

### Example

Write each of the following in simplest power form.

a. $\left(8^5 \times 3^4\right)^3$  b. $\left[(-4)^2 \times 2^3\right]^5$  c. $\left(3^{-4} \times 4^{-2}\right)^{-3}$

### Solution

a. $\left(8^5 \times 3^4\right)^3 = 8^{5\times3} \times 3^{4\times3}$
$= 8^{15} \times 3^{12}$

b. $\left[(-4)^2 \times 2^3\right]^5 = (-4)^{2\times5} \times 2^{3\times5}$
$= (-4)^{10} \times 2^{15}$

c. $\left(3^{-4} \times 4^{-2}\right)^{-3} = 3^{(-4)\times(-3)} \times 4^{(-2)\times(-3)}$
$= 3^{12} \times 4^6$

8. Express each of the following as a product of powers.

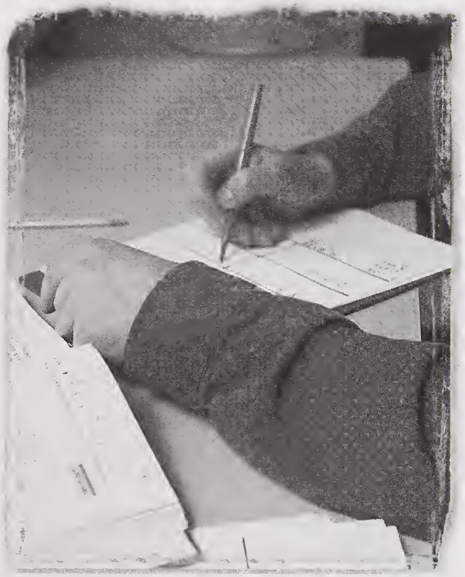
a. $\left(8^{7} \times 6^{2}\right)^{4}$

b. $\left[(-3)^{2} \times(-4)^{7}\right]^{3}$

c. $\left(7 \times 4^{2}\right)^{5}$

d. $\left(0.5^{4} \times 1.4^{3}\right)^{-2}$

Check your answers by turning to the Appendix, page 145.



## Power of a Quotient

Now that you have studied the power of a product, what about the power of a quotient? Is there a similarity? Check it out as you work through the following questions.

9. Complete the following table. The first one is done as an example.

| Expression | Factored Form | Power Form |
|---|---|---|
| $\left(\frac{8^2}{4^3}\right)^3$ | $\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)$ $=\frac{8^2\times 8^2\times 8^2}{4^3\times 4^3\times 4^3}$ | $\frac{8^6}{4^9}$ |
| $\left(\frac{5^4}{3^2}\right)^2$ | | |
| $\left(\frac{4^7}{2^5}\right)^3$ | | |

10. What shortcut method do you see for simplifying the power of a quotient?

Check your answers by turning to the Appendix, page 146.

You have just discovered the rule for the power of a quotient.

**To simplify the power of a quotient, multiply each of the exponents inside the brackets by the exponent outside the brackets.**

## Example

Write each of the following in simplest power form.

**a.** $\left(\dfrac{15^4}{8^5}\right)^3$ **b.** $\left[\dfrac{(-4)^3}{-6}\right]^4$

## Solution

**a.** $\left(\dfrac{15^4}{8^5}\right)^3 = \dfrac{15^{4\times3}}{8^{5\times3}} = \dfrac{15^{12}}{8^{15}}$

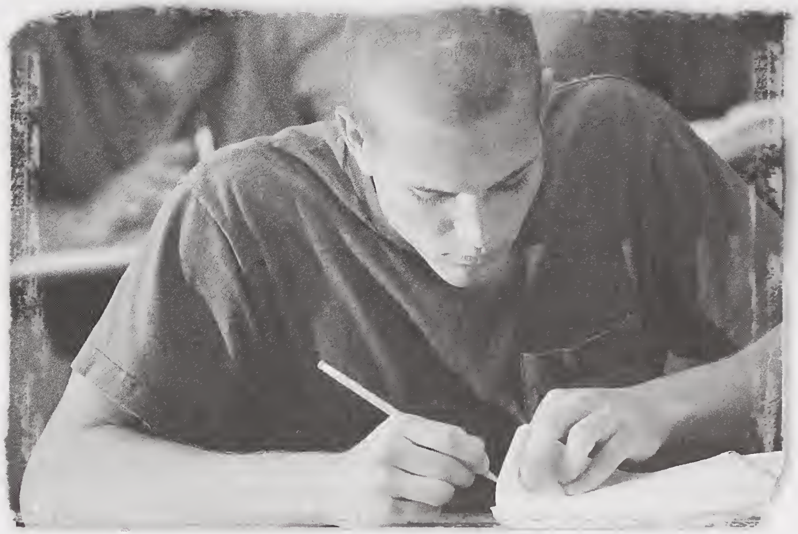
**b.** $\left[\dfrac{(-4)^3}{-6}\right]^4 = \dfrac{(-4)^{3\times4}}{(-6)^{1\times4}} = \dfrac{(-4)^{12}}{(-6)^4}$

**11.** Express each of the following as a quotient of powers.

**a.** $\left(\dfrac{8^4}{5^3}\right)^3$ **b.** $\left(\dfrac{7^4}{3^5}\right)^6$

**c.** $\left(\dfrac{4^7}{3^4}\right)^{-3}$ **d.** $\left[\dfrac{(-5)^{-2}}{(-2)^{-6}}\right]^{-2}$

Check your answers by turning to the Appendix, page 146.



## Non-Numerical Bases

In this activity, you discovered some additional properties or rules for powers using patterns and specific numbers. Now, you are going to generalize those rules by using variables to replace the numbers.

### Example

Express $\left(n^2\right)^3$, where $n$ is any rational number, in simplest power form.

### Solution

$$\begin{aligned}\left(n^2\right)^3 &= n^2 \times n^2 \times n^2 \\ &= n^{2+2+2} \\ &= n^6\end{aligned} \qquad \text{or} \qquad \begin{aligned}\left(n^2\right)^3 &= n^{2\times 3} \\ &= n^6\end{aligned}$$

### Example

Express $\left(n^a\right)^b$, where $n$ is any rational number and $a$ and $b$ are integers, in simplest power form.

### Solution

$$\begin{aligned}\left(n^a\right)^b &= \underbrace{n^a \times n^a \times \ldots \times n^a}_{b \text{ factors}} \\ &= n^{a\times b}\end{aligned}$$

As you can see, the rule is the same as discovered previously, except variables have been used to represent all the possible number replacements.

To simplify a power of a power, multiply the exponents.

$$\left(n^a\right)^b = n^{ab}$$

To simplify a power of a product or a power of a quotient, multiply each of the exponents inside the brackets by the exponent outside the brackets.

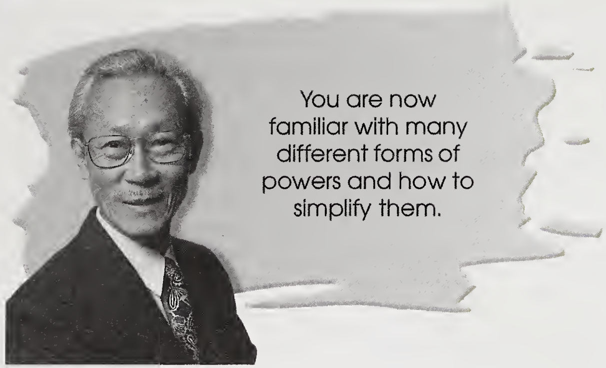
$$\left(n^a \times m^b\right)^c = n^{ac} \times m^{bc}$$
$$\left(n^a \div m^b\right)^c = n^{ac} \div m^{bc}$$

**12.** Simplify the following.

**a.** $\left(a^2\right)^3$ **b.** $\left(y^3\right)^4$ **c.** $\left(m^4 \times n^3\right)^2$

**d.** $\left(a^2 b^2\right)^3$ **e.** $\left(\frac{c^2}{b^3}\right)^2$ **f.** $\left(a^4 \div b^2\right)^3$

Check your answers by turning to the Appendix, page 147.




## Now Try This

Use a problem-solving strategy to answer the following questions.

**13.** **a.** Complete the following. Can you see a pattern in the answers?
**Hint:** Express them as squares.

$1^3 = 1$

$1^3 + 2^3 = 9$

$1^3 + 2^3 + 3^3 =$

$1^3 + 2^3 + 3^3 + 4^3 =$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 =$

**b.** What would be the sum of the first six cubic numbers?

Check your answers by turning to the Appendix, page 147.

## Looking Back

In Activity 3, you extended your knowledge of powers. You discovered rules for a power of a power, a power of a product, and a power of a quotient.

It is common for students to feel like there is a lot to memorize, but that is not the case. Use simple expressions to test the laws.

For example, if you forget if you add or multiply the exponents when you are simplifying $\left(a^2\right)^3$, do a test like the following:

$$\begin{aligned}\left(5^2\right)^3 &= 5^2 \times 5^2 \times 5^2\\ &= (5\times 5)\times(5\times 5)\times(5\times 5)\\ &= 5^6\end{aligned}$$

This test shows that you need to multiply the exponents.

$$\begin{aligned}\therefore\ \left(a^2\right)^3 &= a^{2\times 3}\\ &= a^6\end{aligned}$$

**14.** In your journal, write an example to help you see what to do when you simplify a power of a power, a power of a product, and a power of a quotient.

Check your answers by turning to the Appendix, page 147.

## Activity 4: Order of Operations

Think about activities that require several steps. Doing the steps in a different order could produce a very different result. For example, imagine the results if a person attempted to do the following steps in a different order:

**Step 1:** Get dressed.
**Step 2:** Eat breakfast.
**Step 3:** Start the car.
**Step 4:** Drive to work.

Mathematically, following defined steps is also very important.

For example, Janine, Dustin, and Alissa were given this expression to evaluate:

$$6 \times 4 + 8 \div 2$$

Janine added first, to get $6 \times 12 \div 2$. Next, she multiplied $6 \times 12$ to get 72. Then she divided by 2 to get an answer of 36.

Dustin multiplied $6 \times 4$ first to get 24. Next, he divided 8 by 2 to get 4. Then he added the 24 and 4 to get an answer of 28.

Alissa first multiplied $6 \times 4$ to get 24. Next, she added 8 to get 32. Then she divided by 2 to get an answer of 16.

All three had some logic in their calculations; however, only one evaluated the expression correctly. Whose answer was correct?

Dustin followed the rules for the order of operations to get his answer of 28.

**The rules for the order of operations are the rules that mathematicians have agreed to follow when evaluating a mathematical expression.**

The order is as follows:

- Perform operations in **brackets** first.
- Work with **exponents** next.
- Then do **division** and **multiplication** in order from left to right.
- Finally, do **addition** and **subtraction** in order from left to right.

Remember BEDMAS. This mnemonic will help you recall the order of operations.

**B** $\rightarrow$ **Brackets**
**E** $\rightarrow$ **Exponents**
**D/M** $\rightarrow$ **Division/Multiplication**
**A/S** $\rightarrow$ **Addition/Subtraction**

## Example

Evaluate $18-3^2\times(5-1)$.

### Solution

$$
\begin{aligned}
&18-3^2\times(5-1) && \longleftarrow \text{Brackets}\\
&=18-3^2\times 4 && \longleftarrow \text{Exponents}\\
&=18-9\times 4 && \longleftarrow \text{Multiplication}\\
&=18-36 && \longleftarrow \text{Change subtraction to addition.}\\
&=18+(-36) && \longleftarrow \text{Addition}\\
&=-18
\end{aligned}
$$

## Example

Evaluate $\frac{5^2-7}{2\times 3}$.

### Solution

$$
\begin{aligned}
&\frac{5^2-7}{2\times 3} && \longleftarrow \text{Rewrite the expression using } \div \text{ sign and brackets.}\\
&=(5^2-7)\div(2\times 3) && \longleftarrow \text{Brackets}\\
&=(25-7)\div(2\times 3) && \longleftarrow \text{Brackets}\\
&=18\div 6 && \longleftarrow \text{Division}\\
&=3
\end{aligned}
$$

1. Evaluate the following expressions.

   **a.** $3^3-3\times 2$

   **b.** $2^3-2^2$

   **c.** $2^2+3^2+4^2-(2+3)^2$

   **d.** $(-8)^2-5^2$

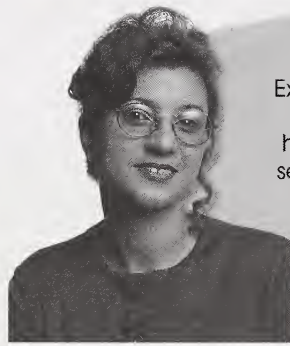
   **e.** $\frac{4^3}{4^2\times 2}$

   **f.** $\frac{9^2-3\times 6}{12-(4+1)}$

   **g.** $\frac{(12-6)\times(12+6)}{(3+3)^2}$

   **h.** $\frac{3^2\times 4+3\times 2}{16\div 4+8+2}$

Check your answers by turning to the Appendix, page 148.



Examine the following examples closely. Notice that when you have a set of brackets within a set of brackets, you must do the inside brackets first.

## Example

Evaluate $10^2 - [3^2 \times (4+1)]$.

## Solution

$$\begin{aligned}
&10^2 - [3^2 \times (4+1)] && \longleftarrow \text{Brackets}\\
&= 10^2 - (3^2 \times 5) && \longleftarrow \text{Brackets}\\
&= 10^2 - (9 \times 5) && \longleftarrow \text{Brackets}\\
&= 10^2 - 45 && \longleftarrow \text{Exponents}\\
&= 100 - 45 && \longleftarrow \text{Subtraction}\\
&= 55
\end{aligned}$$

## Example

Evaluate $6 \div [3^2 - (-2+1)]$.

## Solution

$$\begin{aligned}
&6 \div [3^2 - (-2+1)] && \longleftarrow \text{Brackets}\\
&= 6 \div [3^2 - (-1)] && \longleftarrow \text{Brackets}\\
&= 6 \div [9 - (-1)] && \longleftarrow \text{Brackets}\\
&= 6 \div (9+1) && \longleftarrow \text{Brackets}\\
&= 6 \div 10 && \longleftarrow \text{Division}\\
&= 0.6
\end{aligned}$$

## Example

Evaluate $\left[3^2 + 5 \times (-1)^3\right]^2$.

## Solution

$$\begin{aligned}
&\left[3^2 + 5 \times (-1)^3\right]^2 && \longleftarrow \text{Brackets}\\
&= [9 + 5 \times (-1)]^2 && \longleftarrow \text{Brackets}\\
&= [9 + (-5)]^2 && \longleftarrow \text{Brackets}\\
&= 4^2 && \longleftarrow \text{Exponents}\\
&= 16
\end{aligned}$$

2. Evaluate the following expressions.

   **a.** $-2\left[(-2)^2 + (-3)^2 + (-4)^2\right]$

   **b.** $100 - \left[9^2 - (13+12)\right]$

   **c.** $\left[(-8)^2 - 5\right]^2$

   **d.** $\left[\dfrac{(-3)^2 + (-7)}{-2}\right]^3$

Check your answers by turning to the Appendix, page 148.

Be careful when you use a calculator to evaluate expressions involving more than one operation.

Calculators differ in how they handle the rules for the order of operations. You should refer to the manual for your model of calculator.

Work through the following example.

## Example

**a.** Evaluate the expression $13-2\times 5$ using the rules for the order of operations.

**b.** Using a simple calculator and a scientific calculator, compare the answers you get when you press the following key sequence.

1 3 − 2 × 5 =

## Solution

**a.** $13-2\times 5=13-10$

$=3$

**b.** **Simple Calculator**

1 3 − 2 × 5 =

55.

You get the wrong answer on the simple calculator.

The simple calculator completes the operations in the order they are entered. In this case, the subtraction is completed first, and then the multiplication.

**Scientific Calculator**

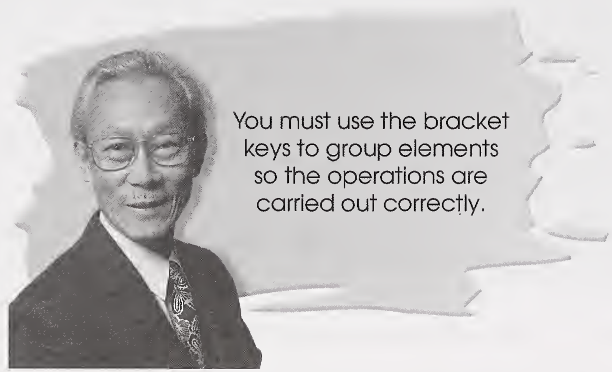
1 3 − 2 × 5 =

3.

You get the correct answer on the scientific calculator.

Scientific calculators are programmed to use the rules for order of operations; simple calculators are not programmed to use these rules.

In some cases, the way the scientific calculator automatically follows the order of operations can lead to incorrect answers.



## Example

Using your calculator, evaluate the expression $\dfrac{4\times12-6^2}{18-(4+2)}$.

## Solution

For these types of expressions, bracket the entire numerator and bracket the entire denominator as they are being entered.

( 4 × 1 2 − 6 $x^2$ )

÷ ( 1 8 − ( 4 + 2 ) ) =

7

Bracketing the entire numerator and the entire denominator instructs the calculator to divide the entire numerator by the entire denominator.

Try evaluating the expression in the preceding example without bracketing the numerator or denominator.

4 × 1 2 − 6 $x^2$

÷ 1 8 + ( 4 + 2 ) =

Because the scientific calculator is programmed to follow the order of operations, it will deal with exponents, and then multiply and divide before it adds or subtracts. Therefore, without brackets when the calculator comes to 6 $x^2$ ÷ 1 8, it will simply divide them in the standard order.

Without bracketing the numerator and denominator, the scientific calculator will give the answer as 40.

As you can see, it is very important to place brackets around the entire numerator and around the entire denominator when evaluating with a calculator.

**3.** Use a scientific calculator to evaluate each of the following expressions.

**a.** $3^3-3\times2$

**b.** $2^3-2^2$

**c.** $2^2+3^2+4^2-(2+3)^2$

**d.** $(-8)^2-5^2$

**e.** $\dfrac{4^3}{4^2\times2}$

**f.** $\dfrac{9^2-3\times6}{12-(4+1)}$

**g.** $\dfrac{(12-6)\times(12+6)}{(3+3)^2}$

**h.** $\dfrac{3^2\times4+3\times2}{16\div4+8+2}$

Check your answers by turning to the Appendix, page 149.

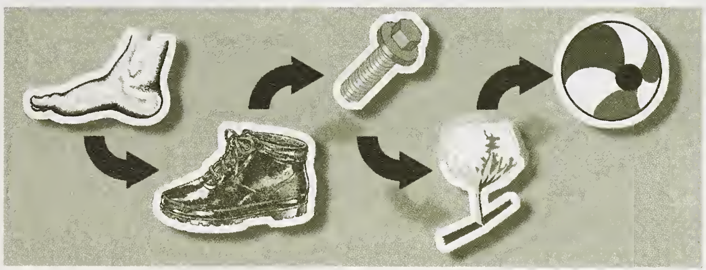
## Now Try This

You are now ready for a problem-solving excursion.

You've probably played the word game called **doublets**. Doublets is a game invented by Lewis Carroll, where the object is to change one word into another with the same number of letters in as few steps as possible. Each step involves changing one letter at a time so that each new combination also forms a real word.

For example, to change FOOT to BALL, you would make the following changes:

FOOT → BOOT → BOLT → BOLL → BALL



Changing FOOT to BALL was done in the minimum number of steps.

4. Write out the steps required to change MATH to TOPS. Try to use as few steps as possible.

Check your answer by turning to the Appendix, page 149.

## Looking Back

In this activity, you used your knowledge of the rules for order of operations to calculate any expression involving a variety of operations directly.

5. Copy the acronym BEDMAS into your journal. Beside each letter of the acronym, write the word the letter represents.

Check your answer by turning to the Appendix, page 150.

# Activity 5: Scientific Notation

Have you ever thought about the number 10? You have 10 fingers and 10 toes. Perhaps that is how the base-10 number system was developed.

The following chart shows several powers of ten.

| Name | Standard Form | Power Form |
|---|---|---|
| one million | 1 000 000 | $10^6$ |
| one hundred thousand | 100 000 | $10^5$ |
| ten thousand | 10 000 | $10^4$ |
| one thousand | 1000 | $10^3$ |
| one hundred | 100 | $10^2$ |
| ten | 10 | $10^1$ |
| one | 1 | $10^0$ |
| one tenth | $\frac{1}{10}$ or 0.1 | $10^{-1}$ |
| one hundredth | $\frac{1}{100}$ or 0.01 | $10^{-2}$ |
| one thousandth | $\frac{1}{1000}$ or 0.001 | $10^{-3}$ |
| one ten thousandth | $\frac{1}{10\,000}$ or 0.0001 | $10^{-4}$ |
| one hundred thousandth | $\frac{1}{100\,000}$ or 0.000 01 | $10^{-5}$ |
| one millionth | $\frac{1}{1\,000\,000}$ or 0.000 001 | $10^{-6}$ |

Very large and very small numbers are sometimes awkward to read and write. Therefore, these numbers are often expressed in scientific notation.

**Scientific notation is a notation for writing very large or very small numbers as a product of a number (between 1 and 10) and a power of 10.**

Work through the following example.

## Example

The distance between Saturn and the Sun is about 1 429 000 000 km. Express this measurement in scientific notation.


NASA

### Solution

**Step 1:** Since this is a very large number, start from the right side of the first non-zero digit and count the number of digits to the decimal point. The total count is the exponent of the power of 10.

1 429 000 000

The count is 9 to the right; therefore, the exponent of the power of 10 is positive 9.

**Step 2:** Write the number in scientific notation form as a product of the number between 1 and 10 and the power of ten.

$$1\,429\,000\,000 = 1.429\times10^{9}$$

The distance from Saturn to the Sun is about $1.429\times10^{9}$ km.

## Example

The mass of an oxygen atom is about 0.000 000 000 000 000 000 026 6 mg. Express this measurement in scientific notation.

## Solution

**Step 1:** Since this is a very small number, start from the right of the first non-zero digit and count the number of digits to the decimal point. The total count is the exponent of the power of 10.

$$0.000\,000\,000\,000\,000\,000\,026\,6$$

The count is 20 to the left; therefore, the exponent is $-20$.

**Step 2:** Write the number in scientific notation form as a product of the number between 1 and 10 and the power of 10.

$$0.000\,000\,000\,000\,000\,000\,026\,6 = 2.66\times10^{-20}$$

The mass of an oxygen atom is about $2.66\times10^{-20}$ mg.

1. Complete each statement so that the number is written in scientific notation.

   a. $24\,000\,000 = 2.4\times10^{\square}$

   b. $0.000\,000\,43 = 4.3\times10^{\square}$

   c. $54\,600\,000\,000 = \square\times10^{10}$

   d. $0.000\,000\,000\,039 = \square\times10^{-11}$

   e. $147\,000\,000 = \square\times10^{\square}$

   f. $0.000\,000\,083 = \square\times10^{\square}$

2. Which of the following numbers are not in scientific notation? Explain. Rewrite them so they are in scientific notation.

   a. $4.8\times2^{10}$ b. $52\times10^{-9}$ c. $7\times10$

   d. $1\times10^{-12}$ e. $0.48\times10^{15}$

3. Write each of the following in scientific notation.

   a. 48 000 000 000 b. 0.000 000 007 2

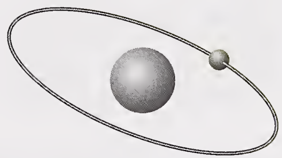
   c. 1 000 000 000 000 000

4. Write these numbers in standard form.

   a. $7.12\times10^{8}$ b. $4.2\times10^{-10}$ c. $1\times10^{5}$

5. The temperature at the centre of the Sun reaches about 13 000 000°C. Write this temperature in scientific notation.

6. The mass of a hydrogen atom is about 0.000 000 000 000 000 000 000 001 67 g. Write this mass in scientific notation.



You may find a scientific calculator very useful when doing multiplication and division with very large or very small numbers. The calculator may present the answers in scientific notation. Use a scientific calculator to answer questions 7 and 8.

7. Enter $187\ 000 \times 567\ 000$ on a scientific calculator. What result do you get?

8. Enter $0.0006 \div 30\ 000$ on a scientific calculator. What result do you get?

Check your answers by turning to the Appendix, page 150.

Scientific calculators express very large and very small numbers in scientific notation. However, these calculators may not display the multiplication sign or the base of the power of 10. For example, the following display means $5.138 \times 10^8$.

$5.138^{08}$

9. Write the following calculator displays in scientific notation.

a. $4.35^{06}$

b. $3.27^{-05}$

10. Why is the answer to question 7 displayed in scientific notation?

11. What is the largest number your scientific calculator can display in standard form?

The answer to question 8 written in standard form is 0.000 000 02. Although this number contains less than the maximum of ten digits, it is written in scientific notation. To discover why, answer questions 12 and 13.

12. Perform the following calculations involving decimals on your calculator, and write out the result in the display.

| | | |
|---|---|---|
| a. $0.03 \div 1$ | b. $0.03 \div 10$ | c. $0.03 \div 100$ |
| d. $0.03 \times 1$ | e. $0.03 \times 0.1$ | f. $0.03 \times 0.01$ |

13. What pattern do you see in the results?

Check your answers by turning to the Appendix, page 151.

## Multiplying Numbers in Scientific Notation

Scientific notation can be used when multiplying very large and very small numbers.

Work through the following example.

### Example

The mass of Earth is about $6.0 \times 10^{24}$ kg. It has been determined that the mass of the Sun is about $3.3 \times 10^{5}$ times greater than the mass of Earth. Calculate the mass of the Sun.

NASA

### Solution

$$\begin{aligned}(6.0 \times 10^{24}) \times (3.3 \times 10^{5}) &= (6.0 \times 3.3) \times (10^{24} \times 10^{5}) \\ &= 19.8 \times 10^{24+5} \\ &= 19.8 \times 10^{29} \\ &= 1.98 \times 10^{1} \times 10^{29} \\ &= 1.98 \times 10^{30}\end{aligned}$$

Therefore, the mass of the Sun is about $1.98 \times 10^{30}$ kg.

**Remember:** When a number is written in scientific notation, it is written as the product of a number (equal to or greater than 1 and less than 10) and a power of 10.

You can use a scientific calculator to multiply numbers in scientific notation directly. To use a scientific calculator directly, you need to be able to enter the numbers in scientific notation.

The easiest method of entering numbers in scientific notation is using the EXP key. This key quickly sets up the $\times 10^{n}$ part of scientific notation.

### Example

Enter into your calculator the scientific notation expression $1.96 \times 10^{12}$.

### Solution

1 • 9 6 EXP 1 2

Note how the EXP represents "times ten to the power of."

### Example

Chemists estimate that $6.02 \times 10^{23}$ molecules of hydrogen gas occupy 24.8 L at 25°C at normal atmospheric pressure. If one molecule of hydrogen gas has a mass of $3.34 \times 10^{-24}$ g, what is the mass of 24.8 L of hydrogen gas?

### Solution

6 • 0 2 EXP 2 3 × 3 • 3 4 × EXP 2 4 +/− =

2.01068

The mass of 24.8 L of hydrogen gas is 2.010 68 g.

**14.** Multiply the following. Write the answer in scientific notation to two decimal places.

**a.** $(3.91 \times 10^{11}) \times (1.84 \times 10^{-19})$

**b.** $(8.14 \times 10^{-4}) \times (9.25 \times 10^{6})$

**c.** $(7.73 \times 10^{-5}) \times (2.68 \times 10^{-2})$

Check your answers by turning to the Appendix, page 152.

## Dividing Numbers in Scientific Notation

Scientific notation can be used when dividing very large and very small numbers.

Work through the following example.

### Example

If 1 200 000 silkworm eggs have a mass of 1000 g, what is the mass of one silkworm egg?

### Solution

$$\frac{1000}{1\,200\,000} = \frac{1 \times 10^{3}}{1.2 \times 10^{6}}$$
$$= (1 \div 1.2) \times (10^{3-6})$$
$$= 0.8\bar{3} \times 10^{-3}$$

This is not scientific notation because $0.8\bar{3}$ is not between 1 and 10.

$$= 8.\bar{3} \times 10^{-1} \times 10^{-3}$$
$$= 8.\bar{3} \times 10^{-1+(-3)}$$
$$= 8.\bar{3} \times 10^{-4}$$

**Remember:** The symbol above the 3 in $8.\bar{3}$ indicates that the 3 keeps repeating to infinity.

The mass of one silkworm egg is $8.\bar{3} \times 10^{-4}$ g.

Complete the following questions.

**15.** Earth's oceans contain many dissolved salts and minerals. There is about 1 kg of gold in $0.25 \text{ km}^3$ of sea water. If the total volume of the oceans is about $1.4 \times 10^9 \text{ km}^3$, how much gold is there in the oceans?

**16.** Light travels at a speed of $1.08 \times 10^9$ km/h. If Saturn is about $1.429 \times 10^9$ km from the Sun, how long does it take for light from the Sun to reach Saturn?

Check your answers by turning to the Appendix, page 152.

## Looking Back

In this activity, you expressed numbers in scientific notation. Then you multiplied and divided numbers in scientific notation.

Answer the following question in your journal.

**17.** What is an advantage of writing very large or very small numbers in scientific notation?

Check your answer by turning to the Appendix, page 152.

## Conclusion

In this section, you worked with exponential notation. You increased your knowledge of powers by examining patterns that helped develop rules for calculating powers. You then used these rules to calculate the solutions to questions and problems involving powers.

The modern colour computer monitor can produce 256 tones of each of the three basic colours. In total, 16 777 216 colours can be produced. Your new knowledge of powers allows you to express this total in exponential form in more than one way, including scientific notation, and show how this total is calculated.

### Assignment

Turn to Assignment Booklet 1C and complete the assignment for Section 3.

## Module Summary

In this module, you increased your knowledge of the structure and properties of numbers. You listed the multiples and factors of a number, found the least common multiple and greatest common factor of two or more numbers. These are skills you will need when you work with fractions in Module 2 and factoring in Module 4. You developed a sense for integers. You compared and ordered integers and performed operations on integers. You discovered ways to express repeated multiplication using exponential notation.

When you first started to read, you learned the properties of letters—how they sounded individually and how they sounded grouped together. These were the building blocks for learning how to read.

Likewise, factors, multiples, integers, and exponents are the building blocks for mathematics. As you continue through Mathematics Preparation 10, you will see how these topics are the foundation for later modules.

# Appendix

Problem-Solving Strategies
Glossary
Suggested Answers
Credits
Cut-out Learning Aids

# Problem-Solving Strategies

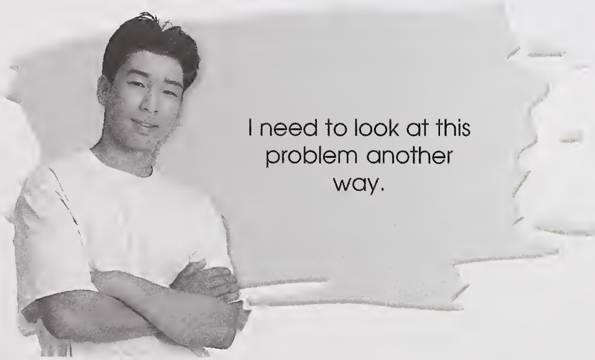
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagram, graph, or chart that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

## Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not successful, it is often helpful to try to change your point of view.

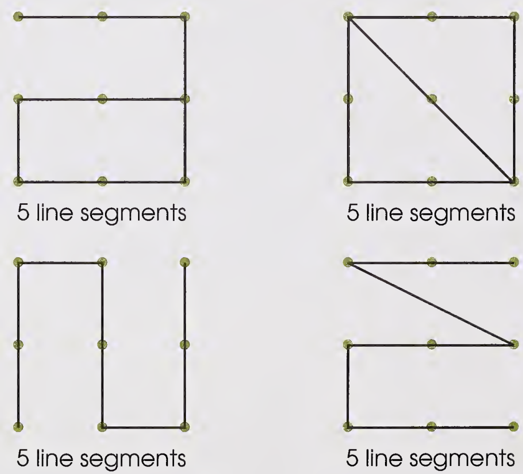



### Example

Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

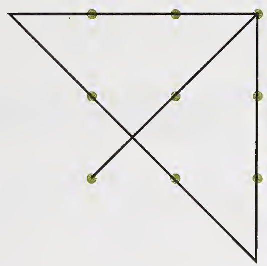
### Solution

**Step 1:** You make several attempts at solving the problem.



**Step 2:** You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem, you must change your point of view and realize that the line segments may lie outside the confines of the dots.



4 line segments

## Using Objects

Objects can be used to help you solve a problem.

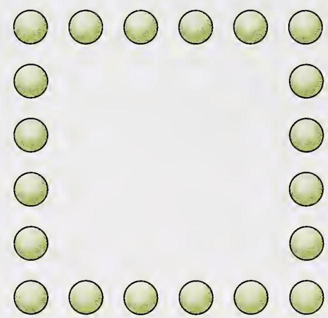
### Example

To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?

### Solution

**Step 1:** Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



**Step 2:** Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

**Step 3:** Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

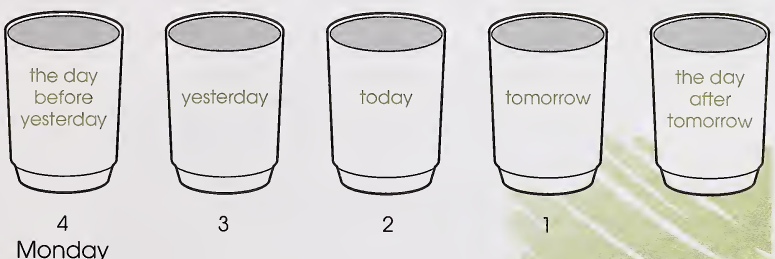
## Example

Four days before the day after tomorrow was Monday. What day of the week is it today?

## Solution

**Step 1:** Use paper cups to represent the days. The cups could be labelled as shown.


**Step 2:** Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.




If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

# Using Diagrams

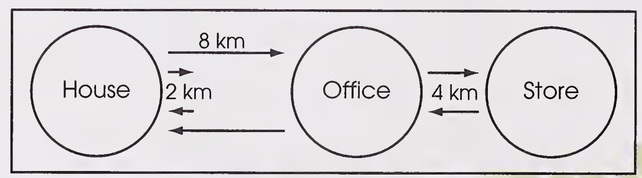
You can use sketches to solve problems.

## Example

Chris's office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon, Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day, he drove straight home. How far did he drive yesterday?

## Solution

**Step 1:** Draw and label a diagram to help you understand the problem.




**Step 2:** Calculate the distance Chris drove.

$$2 + 2 + 8 + 4 + 4 + 8 = 28$$

Chris drove 28 km.

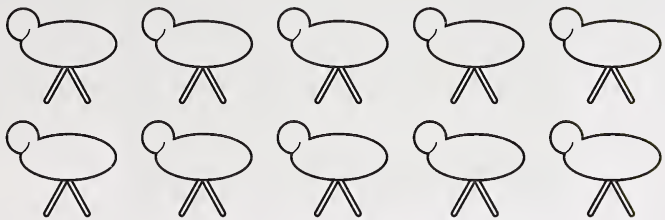
### Example

There are 10 animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

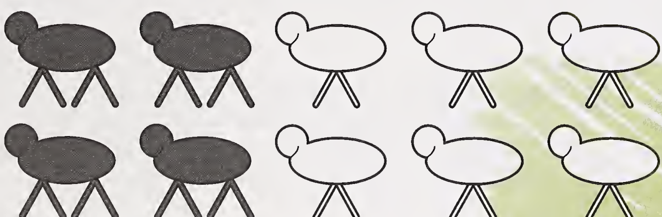
### Solution

Draw a diagram to help you solve the problem.

**Step 1:** Every animal has at least 2 legs. So, draw a diagram of 10 animals, each of which has at least 2 legs.



**Step 2:** The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.

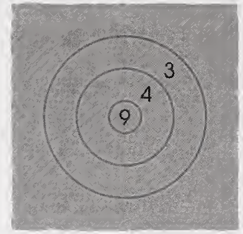


There are 4 sheep and 6 chickens.

## Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

### Example




Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.

If Julio throws three darts and all three hit the target, how many different point totals are possible?

### Solution

Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score

$9+9+4=22$

$9+9+3=21$

$9+4+4=17$

$9+4+3=16$

$9+3+3=15$

$4+4+4=12$

$4+4+3=11$

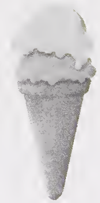
$4+3+3=10$

$3+3+3=9$ ← lowest possible score

There are 10 possible point totals.

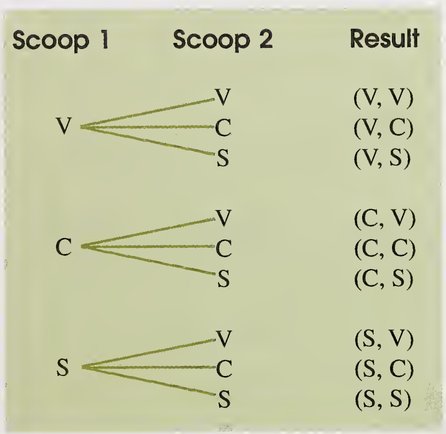
## Example

At an ice-cream parlour there are 3 flavours of ice cream: chocolate, vanilla, and strawberry. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate one. Order makes a difference.



### Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.




Nine different double-scoop ice-cream cones are possible.

## Using Venn Diagrams

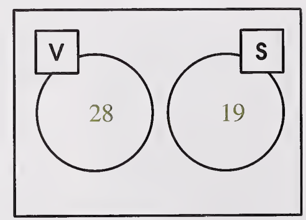
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician by the name of John Venn (1834–1923).

## Example

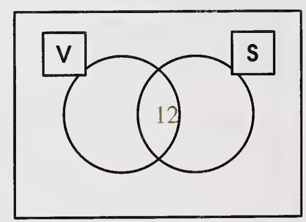
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

### Solution

**Step 1:** Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



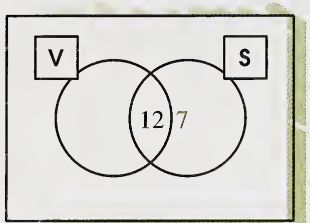

**Step 2:** Represent the members of both teams by the intersection of the two circles.



**Step 3:** Now, calculate the number of girls who belong only to the swim team.

$19 - 12 = 7$

So, 7 girls belong only to the swim team.

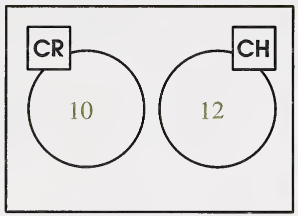



## Example

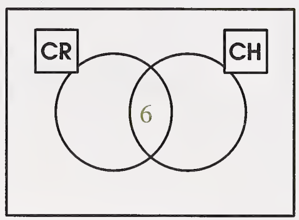
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

### Solution

**Step 1:** The problem states that 10 plates were cracked (CR) and 12 plates were chipped (CH). Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

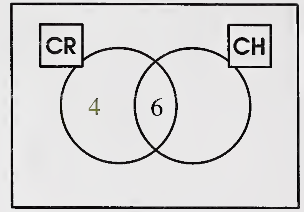



**Step 2:** The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



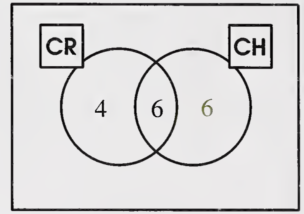

**Step 3:** Calculate the number of plates that were only cracked.

$10 - 6 = 4$

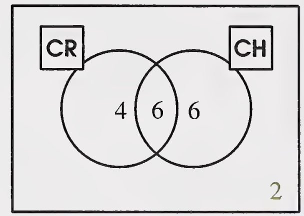



**Step 4:** Calculate the number of plates that were only chipped.

$12 - 6 = 6$



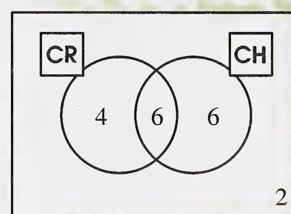

**Step 5:** The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.




**Step 6:** Calculate the total number of plates that were bought.

$4 + 6 + 6 + 2 = 18$

So, 18 plates were bought.



## Example

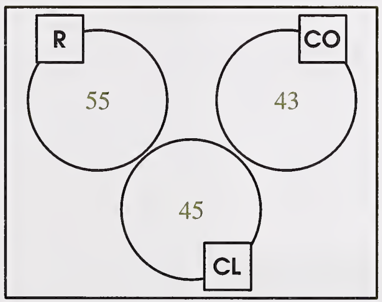
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

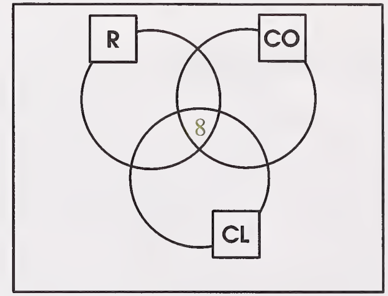
How many people do not like any of these forms of music?

## Solution

**Step 1:** The problem states that 55 people like rock music (R), 43 like country music (CO), and 45 like classical music (CL). Use three circles to represent these preferences.

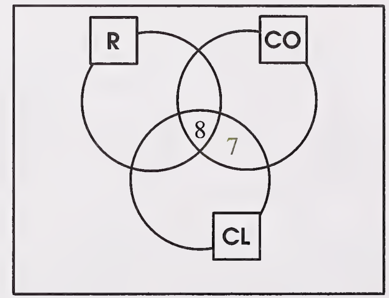



**Step 2:** The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



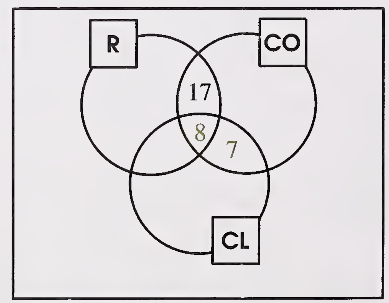

**Step 3:** The problem states that 15 people like both country and classical music. Calculate the number of people who like country and classical music, but not rock music.

$$15 - 8 = 7$$



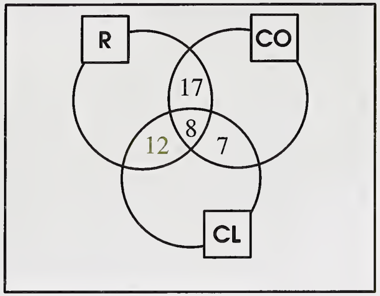

**Step 4:** The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but not classical music.

$$25 - 8 = 17$$



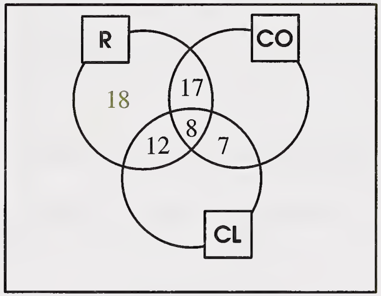
**Step 5:** The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20-8=12$$



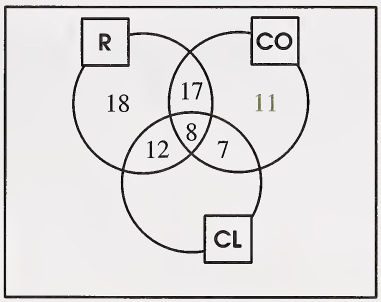

**Step 6:** Now that you know the number of people who like rock music and any other form of music, calculate the number of people who only like rock.

$$55-(17+8+12)=18$$



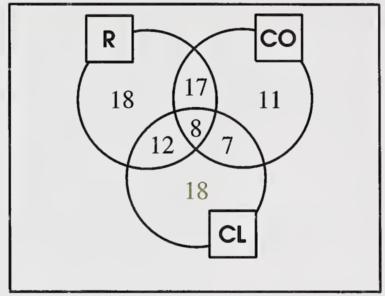

**Step 7:** Now that you know the number of people who like country music and any other form of music, calculate the number of people who only like country music.

$$43-(17+8+7)=11$$



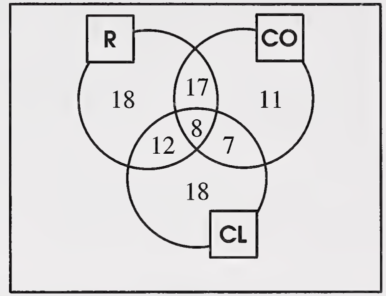

**Step 8:** Now that you know the number of people who like classical music and any other form of music, calculate the number of people who only like classical music.

$$45-(12+8+7)=18$$



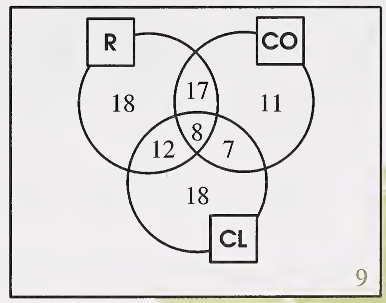

**Step 9:** Now, you can calculate the total number of people who like rock, country, or classical music.

$$18+17+8+12$$
$$+11+7+18=91$$




**Step 10:** The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100-91=9$$




Of the 100 people surveyed, 9 people do not like any of these forms of music.

## Making a Table

You can use a table to organize information.

### Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many bags did she pack and how many customers did she assist that day?

### Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

In the first column, put 1 bag packed and 18 customers assisted. Calculate the money received for that.

$$\begin{aligned} 1 \times 35¢ &= \$\ .35 \\ \underline{18 \times 20¢} &= \underline{\$3.60} \\ \text{Total}\quad & \$3.95 \end{aligned}$$

Then complete the table for other combinations.

| Packing | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Assisting | 18 | 17 | 16 | 15 | 14 | 13 | 12 |
| Earned | $3.95 | $4.10 | $4.25 | $4.40 | $4.55 | $4.70 | $4.85 |

correct guess

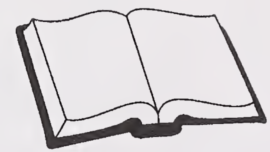
Hannah packed 7 bags and assisted 12 customers that day.

## Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

### Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.



### Solution

**Method 1: Using Paper and Pencil**

**Step 1:** Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

**Step 2:** Make a guess; check your guess; and revise if necessary.

It is helpful to organize your guesses in a table.

| | | | |
|---|---|---|---|
| **Left-Hand Page Number** | 50 | 52 | 54 |
| **Right-Hand Page Number** | 51 | 53 | 55 |
| **Product of Page Numbers** | 2550 | 2756 | 2970 |

correct guess (2970)

So, the book was opened to pages 54 and 55.

**Method 2: Using a Computer**

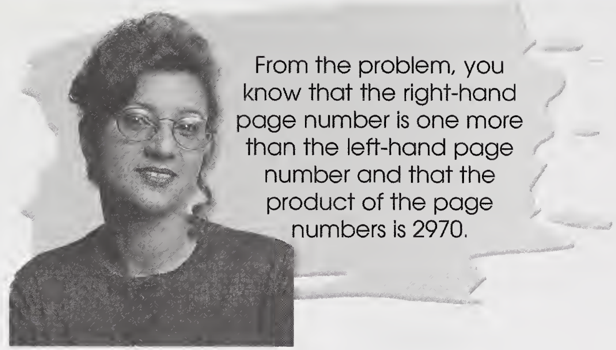
Use a computer and a spreadsheet program to help you solve the problem.

**Step 1:** Label the cells in the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead, you use the entry bar at the top of the spreadsheet.

A5 | ✗ | ✓ | Product of page numbers

| | A | B |
|---|---|---|
| 1 | **Book Problem** | |
| 2 | | |
| 3 | Left-hand page number | |
| 4 | Right-hand page number | |
| 5 | Product of page numbers | |
| 6 | | |

**Step 2:** Enter either a number or a formula for each of the cells.

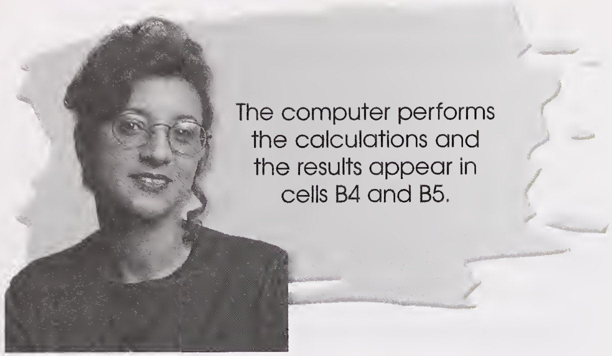



Here is what you enter:

- Begin with cell B3 and enter a guess of 50.
- Next, enter "=B3+1" in cell B4. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter "=B3*B4" in cell B5. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 | ✗ | ✓ | =B3*B4

| | A | B |
|---|---|---|
| 1 | **Book Problem** | |
| 2 | | |
| 3 | Left-hand page number | 50 |
| 4 | Right-hand page number | 51 |
| 5 | Product of page numbers | 2550 |
| 6 | | |




**Step 3:** Check your initial guess, and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter a guess of 54.

B3 | ✗ | ✓ | 54

| | A | B |
|---|---|---|
| 1 | **Book Problem** | |
| 2 | | |
| 3 | Left-hand page number | 54 |
| 4 | Right-hand page number | 55 |
| 5 | Product of page numbers | 2970 |
| 6 | | |

The computer performs the calculations and different results appear in cells B4 and B5.

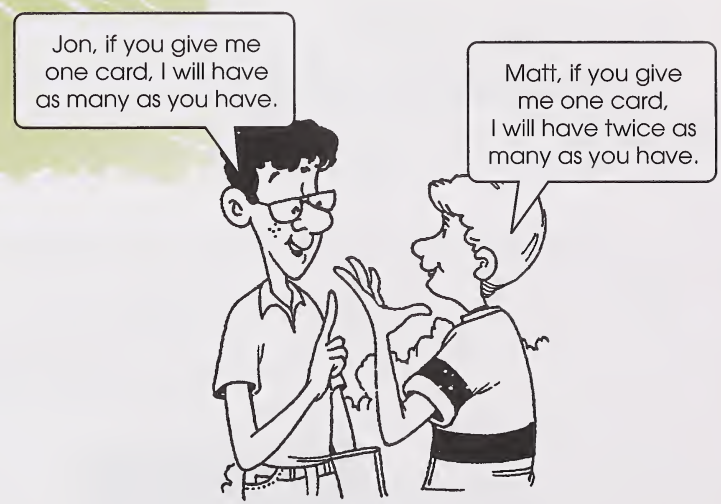
**Step 4:** Check your initial guess, and revise it if necessary. The product is 2970.

So, the book was opened to pages 54 and 55.

## Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

### Example




How many cards does each boy have?

### Solution

**Step 1:** Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.

- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

**Step 2:** Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

| | | | | | |
|---|---|---|---|---|---|
| **Matt's cards** | 1 | 2 | 3 | 4 | 5 |
| **Jon's cards** | 3 | 4 | 5 | 6 | 7 |

correct guess

So, Matt has 5 cards and Jon has 7 cards.

**Note**

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange, 6.

If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange. (Jon will have 8 cards and Matt will have 4 cards.)

## Working Backwards

In mathematics, you are usually given information and then asked to find the answer. Sometimes, however, you are given the answer and then asked to find a piece of information. In cases like these, you need to work backwards.

### Example

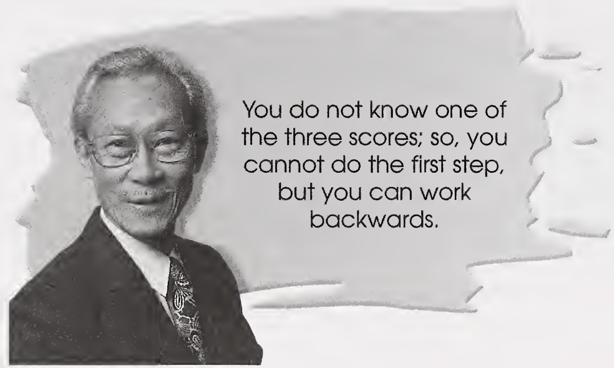
Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What must he score in his third game to maintain his average?

### Solution

**Step 1:** Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.

Start ? → Add 195. → Add 175. → Divide by 3. → End 198

You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.



**Step 2:** Use a reverse flow chart to work backwards.

These operations undo operations in the original flow chart.

End ? ← Subtract 195. ← Subtract 175. ← Multiply by 3. ← Start 198

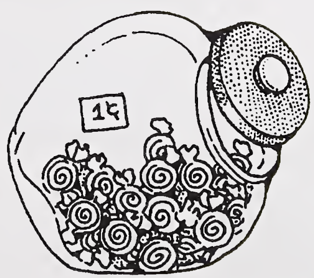
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the score Ross must get in the third game.

Ross must score 224 in the last game to maintain his average.

**Note:** You can check to see if 224 is the correct third score by using the flow chart in Step 1.

## Example

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?




## Solution

**Step 1:** Use a flow chart to show the order of operations.

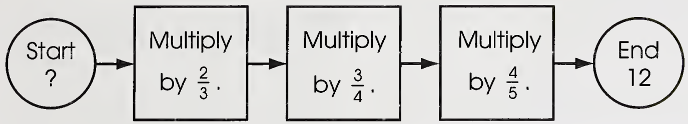
Start ? → Multiply by $\frac{1}{3}$. → Subtract ? → Multiply by $\frac{1}{4}$. → Subtract ? → Multiply by $\frac{1}{5}$. → Subtract ? → End 12

This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

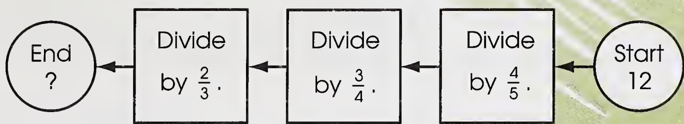
Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds. $1-\frac{1}{3}=\frac{2}{3}$
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths. $1-\frac{1}{4}=\frac{3}{4}$
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths. $1-\frac{1}{5}=\frac{4}{5}$

The following flow chart is more useful.




**Step 2:** Use a reverse flow chart to work backwards.




Ruth had 30 candies at the beginning.

## Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

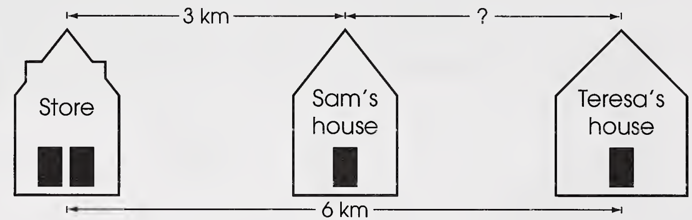
### Example

The average distance from the Sun to Mars is 228 000 000 km, and the average distance from the Sun to Earth is 150 000 000 km. What is the average distance between Earth and Mars?

### Solution

**Step 1:** If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?




$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

**Step 2:** Once you understand the related but simpler problem, you can solve the original problem.

$$228\ 000\ 000 - 150\ 000\ 000 = 78\ 000\ 000$$

The average distance from Earth to Mars is 78 000 000 km.

## Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

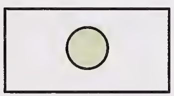
### Example

There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

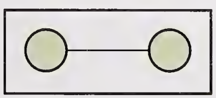
### Solution

**Step 1:** Try solving the problem with 1, 2, 3, 4, or 5 students.

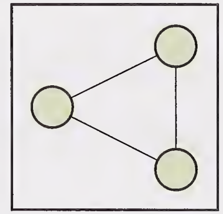
- If there is 1 student, no handshakes can be exchanged (as illustrated in the following diagram).



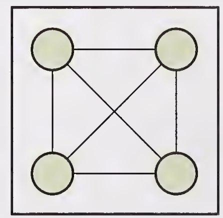
- If there are 2 students, 1 handshake can be exchanged (as illustrated in the following diagram).



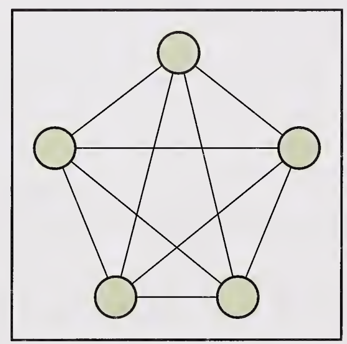
- If there are 3 students, 3 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 4 students, 6 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 5 students, 10 handshakes can be exchanged (as illustrated in the following diagram).



**Step 2:** Look for a pattern.

| People | Handshakes | Pattern |
|---|---|---|
| 1 | 0 | |
| 2 | 1 | +1 |
| 3 | 3 | +2 |
| 4 | 6 | +3 |
| 5 | 10 | +4 |

**Step 3:** Apply the pattern.

A calculator is helpful for applying the pattern. Go up to 19, because each student will shake the hand of 19 others.

1 + 2 + 3 + 4 + 5 + 6 +
7 + 8 + 9 + 10 + 11 +
12 + 13 + 14 + 15 +
16 + 17 + 18 + 19 =

190.

There are a total of 190 handshakes exchanged.

## Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

### Example

Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

### Solution

**Step 1:** Decide what clues are given. The clues indicate the following:

- The donkey and the coyote did not finish first because they did not beat the lion.
- The rabbit did not finish first because the coyote beat it.
- The ostrich was faster than the coyote.
- The rabbit was faster than the donkey.
- The ostrich did not finish first because it was not the fastest.

The clues can be shown on a table.

| | Donkey | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | |
| Second | | | | | |
| Third | | | | | |
| Fourth | | | | | |
| Fifth | | | | | |

**Step 2:** Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now, reconsider the clues:

- The rabbit did not finish second because the coyote beat it.
- The coyote did not finish second because the ostrich was faster.
- The donkey did not finish second because it lagged behind the rabbit.

The additional clues can be shown in the table.

| | Donkey | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | ✓ |
| Second | ✗ | ✗ | ✗ | | ✗ |
| Third | | | | | ✗ |
| Fourth | | | | | ✗ |
| Fifth | | | | | ✗ |

**Step 3:** Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now, reconsider the clues:

- The rabbit did not finish third because the coyote beat it.
- The donkey did not finish third because it lagged behind the rabbit.

The additional clues can be shown in the table.

| | Donkey | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | ✓ |
| Second | ✗ | ✗ | ✗ | ✓ | ✗ |
| Third | ✗ | | ✗ | ✗ | ✗ |
| Fourth | | | | ✗ | ✗ |
| Fifth | | | | ✗ | ✗ |

**Step 4:** Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now, reconsider the clues.

Because the donkey lagged behind the rabbit, the rabbit was fourth and the donkey was fifth.

| | Donkey | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | ✓ |
| Second | ✗ | ✗ | ✗ | ✓ | ✗ |
| Third | ✗ | ✓ | ✗ | ✗ | ✗ |
| Fourth | ✗ | ✗ | ✓ | ✗ | ✗ |
| Fifth | ✓ | ✗ | ✗ | ✗ | ✗ |

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

## Using Truth Tables

A strategy you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

### Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

### Solution

**Step 1:** Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

| Charlie Chipmunk | Sammy Squirrel | Harold Hare |
|---|---|---|
| F | | |

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

| Charlie Chipmunk | Sammy Squirrel | Harold Hare |
|---|---|---|
| F | **T** | |

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

| Charlie Chipmunk | Sammy Squirrel | Harold Hare |
|---|---|---|
| F | T | **T** |

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

**Step 2:** Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

| Charlie Chipmunk | Sammy Squirrel | Harold Hare |
|---|---|---|
| F | T | T |
| **T** | | |

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

| Charlie Chipmunk | Sammy Squirrel | Harold Hare |
|---|---|---|
| F | T | T |
| T | **F** | |

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

| Charlie Chipmunk | Sammy Squirrel | Harold Hare |
|---|---|---|
| F | T | T |
| T | F | **F** |

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy Squirrel was the robber.

## Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

### Example

Twelve less than a number is $-3$. Find the number.

### Solution

**Step 1:** Let the unknown be equal to a variable. Let the number be $n$.

**Step 2:** Write an equation.

$$n-12=-3$$

Twelve less than a number is $-3$.

**Step 3:** Solve the equation.

$$\begin{array}{rcr} n-12 &=& -3 \\ +12 & & +12 \\ \hline n &=& 9 \end{array}$$

The number is 9.

### Example

The second of two numbers is 7 times the first. The sum of the two numbers is 32. Find the numbers.

### Solution

**Step 1:** Write statements for the unknowns. Use only one variable.

Let $n$ be the first number and let $7n$ be the second number.

The second of two numbers is 7 times the first.

**Step 2:** Write an equation.

$$n+7n=32$$

The sum of the two numbers is 32.

**Step 3:** Solve the equation.

$$\begin{aligned} n+7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

**Step 4:** Find the second number if the first number is 4.

$$\begin{aligned} 7n &= 7\times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

## Example

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is \$8.50, how many coins of each type does she have?

## Solution

**Step 1:** Write statements for the unknowns. Use only one variable.

Let the number of dimes be $n$.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

| Type of Coin | Number of Coins | Value in Cents |
|---|---|---|
| Dimes | $n$ | $10n$ |
| Nickels | $n + 20$ | $5(n + 20)$ |
| Total | | 850 |

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

**Step 2:** Write an equation.

$$5(n+20)+10n=850$$

**Step 3:** Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 15n+100 &= 850 \\ -100 \quad & \quad -100 \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

**Step 4:** Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} n+20 &= 50+20 \\ &= 70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

# Glossary

**absolute value:** the value of an integer without regard to the sign; also called the magnitude

**additive inverses:** two numbers whose sum is 0

**base:** (of a power) the part of a power that shows what factor is multiplied by itself

e.g., In $3^5$, the base is 3.

**Cartesian coordinate system:** a method of defining the position of a point in two-dimensional space

**common factor:** a factor of two or more numbers

**common multiple:** a multiple of two or more numbers

**composite number:** a number that has more than two different factors

**consecutive numbers:** integers that increase in order by 1

$-3, -2, -1$ or 101, 102, 103, 104

**denominator:** the term below the line in the fraction

**dividend:** in division, the number you are dividing

**dividing:** the process of finding how many groups of a given size are continued in a given set of elements; the process of finding how many elements there are in a given number of groups, where each group has the same number of elements

**divisibility test:** a test to quickly decide if a number is a factor of another number

**divisible:** able to be divided so there is no remainder

**divisor:** in division, the number you are dividing by; also called a **factor**

**exponent:** (of a power) the part of a power that shows how many times the base is used as a factor

e.g., In $3^5$, the exponent is 5.

**exponential form:** a number written in the form $a^b$

**factor:** (noun) any one of the numbers used in multiplication to form a product; a whole number that exactly divides another whole number

**factor:** (verb) to express a number as a product of its factors

**factored form:** (of a number) a form of a number written as the product of factors

$3^4 = 3\times3\times3\times3$

**factor tree:** a diagram used to find the prime factorization of a number

**greatest common factor (GCF):** the greatest factor of two or more numbers

**integer:** a number in the set $\{\ldots -3, -2, -1, 0, 1, 2, \ldots\}$

**least common denominator:** the least common multiple of the denominators

**least common multiple (LCM):** is the least multiple of two or more numbers

**multiple:** (of a number) the product of the number and a natural number; the sum of adding the same number repeatedly

**multiplier:** in multiplication, the number you are multiplying by

**multiplying:** the process of finding the total number of elements in a given number of groups where each group has the same number of elements

**natural number:** a number in the set $\{1, 2, 3, \ldots\}$, also called the counting numbers

**opposite integer:** an integer having the same absolute value but a different sign; also called an additive inverse

**order of operations:** the order in which operations are performed: brackets, then exponents, then multiplication or division, and then addition or subtraction

**power:** a compact way of showing the product of identical factors; consists of a base and an exponent

**power form:** a form of a number written as a power; sometimes called **exponential form**

**prime factor:** a factor that is a prime number

**prime factorization:** the process of finding all the prime factors of a given number; an expression of a number as the product of its prime factors

**prime number:** a number that has exactly two factors (the number 1 and itself)

**problem:** a task for which the method of finding the answer (as well as the answer itself) is not immediately known

**product:** the result of multiplication

**proper divisor:** (of a number) a factor less than the number itself; also called a **proper factor**

**proper factor:** (of a number) a factor less than the number itself; also called a **proper divisor**

**quadrant:** one of the four regions formed by the intersection of the $x$- and $y$-axes

**quotient:** the result of division

**reciprocal:** (of a number) is the number whose product with the given number is equal to 1.

e.g., The reciprocal of 4 is $\frac{1}{4}$.

**relatively prime:** two numbers for which the greatest common factor is 1

**remainder:** in division, the amount left over

**T-table:** a table made in the shape of the letter T

**whole number:** a number in the set $\{0, 1, 2, 3, \ldots\}$

# Suggested Answers

## Section 1: Activity 1

1. a. 4, 8, 12, 16, 20
   b. 5, 10, 15, 20, 25

2. a. 88, 96, 104, 112, 120
   b. 84, 96, 108, 120, 132
   c. 92, 115, 138, 161, 184
   d. 96, 128, 160, 192, 224

3. a. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
   b. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
   c. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60

      9, 18, 27, 36, 45, 54, 63, 72, 81, 90

   d. 6, 12, $\boxed{18}$, 24, 30, 36, 42, 48, 54, 60

      9, $\boxed{18}$, 27, 36, 45, 54, 63, 72, 81, 90

4. a. 4, 8, $\boxed{12}$, 16, 20, 24, 28, 32, 36, …

      6, $\boxed{12}$, 18, 24, 30, 36, …

      The first three common multiples are 12, 24, and 36. The least common multiple is 12.

   b. 8, 16, $\boxed{24}$, 32, 40, 48, 56, 64, 72, 80, …

      12, $\boxed{24}$, 36, 48, 60, 72, 84, …

      The first three common multiples are 24, 48, and 72. The least common multiple is 24.

   c. 18, 36, 54, $\boxed{72}$, 90, 108, 126, 144, 162, 180, 198, 216, …

      24, 48, $\boxed{72}$, 96, 120, 144, 168, 192, 216, …

      The first three common multiples are 72, 144, and 216. The least common multiple is 72.

5. a. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, $\boxed{60}$
      6, 12, 18, 24, 30, 36, 42, 48, 54, $\boxed{60}$
      10, 20, 30, 40, 50, $\boxed{60}$

      The least common multiple is 60.

   b. 16, 32, 48, 64, 80, 96, 112, 128, $\boxed{144}$
      18, 36, 54, 72, 90, 108, 126, $\boxed{144}$
      24, 48, 72, 96, 120, $\boxed{144}$

      The least common multiple is 144.

6. a. **Camilla:** 9, 18, 27, $\boxed{36}$
**Karen:** 12, 24, $\boxed{36}$

The least amount of time in which Camilla and Karen could cross the finish line together is 36 min.

b. **Camilla:** $36 \div 9 = 4$
**Karen:** $36 \div 12 = 3$

Camilla will have completed 4 laps and Karen will have completed 3 laps. So, Camilla will have completed more laps in 36 min.

Camilla will have completed $4 - 3 = 1$ more lap than Karen.

7. a. **Plant 1:** 8, 16, $\boxed{24}$, 32, 40
**Plant 2:** 12, $\boxed{24}$, 36, 48, 60

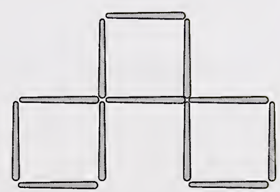
Every 24 days, both plants will be watered on the same day. So, Ming will have to water both plants on January 24.

b. $365 \div 24 = 15$

Both plants will be watered on the same day 15 times in the year.

## Now Try This

8. You may use diagrams or actual toothpicks to help solve this problem.



9. You can make an organized list of the possible combinations of coins that will add up to $0.80. The list may be organized in a table.

| Quarters | Dimes | Nickels |
|---|---|---|
| 0 | 0 | 16 |
| 0 | 1 | 14 |
| 0 | 2 | 12 |
| 0 | 3 | 10 |
| 0 | 4 | 8 |
| 0 | 5 | 6 |
| 0 | 6 | 4 |
| 0 | 7 | 2 |
| 0 | 8 | 0 |
| 1 | 0 | 11 |
| 1 | 1 | 9 |
| 1 | 2 | 7 |
| 1 | 3 | 5 |
| 1 | 4 | 3 |
| 1 | 5 | 1 |
| 2 | 0 | 6 |
| 2 | 1 | 4 |
| 2 | 2 | 2 |
| 2 | 3 | 0 |
| 3 | 0 | 1 |

10. The least common multiple (LCM) of two numbers is not always the product of the two numbers. For example, the LCM of 3 and 6 is 6, while the product is 18. Another example is 8 and 12.

Multiples for 8 are 8, 16, 24, 32, 40, …

Multiples for 12 are 12, 24, 36, 48 , 60, …

The LCM of 8 and 12 is 24, while the product of 8 and 12 is 96.

**11.** **a.** The denominators are 3 and 9. Find the multiples of each.

3, 6, 9, 12, …

9, 18, 27, …

The LCD is 9.

**b.** The denominators are 4 and 18.

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, …

18, 36, 54, …

The LCD is 36.

**c.** The denominators are 7 and 12.

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, …

12, 24, 36, 48, 60, 72, 84, …

The LCD is 84.

## Looking Back

**12.** The first nine multiples of 9, starting with 9, are 9, 18, 27, 36, 45, 54, 63, 72, and 81.

For each multiple, when you add the number in the tens place to the number in the ones place, the sum is 9.

## Section 1: Activity 2

**1.** **a.**

$$\begin{array}{r} 13 \\ 7\overline{)94} \\ \underline{7\phantom{4}} \\ 24 \\ \underline{21} \\ 3 \end{array}$$

9 4 ÷ 7 =

13.42857143

Because the quotient is not a whole number (or because there is a remainder), 94 is not a multiple of 7.

**b.**

$$\begin{array}{r} 21 \\ 4\overline{)86} \\ \underline{8\phantom{6}} \\ 06 \\ \underline{4} \\ 2 \end{array}$$

8 6 ÷ 4 =

21.5

No, 86 is not a multiple of 4.

**c.**

$$\begin{array}{r} 8 \\ 12\overline{)98} \\ \underline{96} \\ 2 \end{array}$$

9 8 ÷ 1 2 =

8.166666667

No, 98 is not a multiple of 12.

d. $14\overline{)56}$ quotient 4

$$\begin{array}{r} 4 \\ 14\overline{)56} \\ \underline{56} \\ 0 \end{array}$$

5 6 ÷ 1 4 =

4.

Yes, 56 is a multiple of 14.

e.

$$\begin{array}{r} 5 \\ 7\overline{)35} \\ \underline{35} \\ 0 \end{array}$$

3 5 ÷ 7 =

5.

Yes, 35 is a multiple of 7.

f.

$$\begin{array}{r} 8 \\ 6\overline{)49} \\ \underline{48} \\ 1 \end{array}$$

4 9 ÷ 6 =

8.166666667

No, 49 is not a multiple of 6.

2. a.

$$\begin{array}{r} 435 \\ 3\overline{)1306} \\ \underline{12\phantom{0}} \\ 10 \\ \underline{9} \\ 16 \\ \underline{15} \\ 1 \end{array}$$

1 3 0 6 ÷ 3 =

435.3333333

No, 3 is not a factor of 1306.

b.

$$\begin{array}{r} 3286 \\ 3\overline{)9858} \\ \underline{9\phantom{000}} \\ 08 \\ \underline{6} \\ 25 \\ \underline{24} \\ 18 \\ \underline{18} \\ 0 \end{array}$$

9 8 5 8 ÷ 3 =

3286.

Yes, 3 is a factor of 9858.

c.

$$\begin{array}{r} 1947 \\ 3\overline{)5842} \\ \underline{3\phantom{000}} \\ 28 \\ \underline{27} \\ 14 \\ \underline{12} \\ 22 \\ \underline{21} \\ 1 \end{array}$$

5 8 4 2 ÷ 3 =

1947.333333

No, 3 is not a factor of 5842.

3. a. no b. no c. yes

4. a. yes b. yes c. no

5. a. yes b. yes c. no

6. a. no b. no c. yes

| | | | | | | |
|---|---|---|---|---|---|---|
| **7.** | **a.** yes | **b.** no | **c.** yes | **d.** yes | **e.** no | **f.** no |
| **8.** | **a.** yes | **b.** no | **c.** yes | **d.** no | **e.** yes | **f.** yes |
| **9.** | **a.** no | **b.** yes | **c.** yes | | | |
| **10.** | **a.** no | **b.** yes | **c.** no | | | |
| **11.** | **a.** yes | **b.** yes | **c.** no | | | |

## Now Try This

**12.** You can solve the problem by finding and applying a pattern.

| Number of Students | Number of Games | Pattern |
|---|---|---|
| 1 | 0 | |
| 2 | 1 | +1 |
| 3 | 3 | +2 |
| 4 | 6 | +3 |
| 5 | 10 | +4 |
| 6 | 15 | +5 |
| 7 | 21 | +6 |
| 8 | 28 | +7 |

In the tournament, 28 games of chess will be played.

## Looking Back

**13.** To be divisible by 4, the last 2 digits must be divisible by 4. The number will be of the form 1 ___ ___ ___, where the last 2 digits are 04, 08, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, or 96.

One such example is 1560. You know this number is divisible by 4 because 60 is divisible by 4.

$$\begin{array}{r} 390 \\ 4\overline{)1560} \\ \underline{12}\phantom{00} \\ 36\phantom{0} \\ \underline{36}\phantom{0} \\ 0 \end{array}$$

Two factors of 1560 are 4 and 390.

## Section 1: Activity 3

**1. a.**

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

**b.**

| 32 | |
|---|---|
| 1 | 32 |
| 2 | 16 |
| ~~3~~ | |
| 4 | 8 |
| ~~5~~ | |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | ~~4~~ |

The factors of 32 are 1, 2, 4, 8, 16, and 32.

c.

| 45 | |
|---|---|
| 1 | 45 |
| ~~2~~ | |
| 3 | 15 |
| ~~4~~ | |
| 5 | 9 |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | ~~5~~ |

The factors of 45 are 1, 3, 5, 9, 15, and 45.

d.

| 70 | |
|---|---|
| 1 | 70 |
| 2 | 35 |
| ~~3~~ | |
| ~~4~~ | |
| 5 | 14 |
| ~~6~~ | |
| 7 | 10 |
| ~~8~~ | |
| ~~9~~ | |
| ~~10~~ | ~~7~~ |

The factors of 70 are 1, 2, 5, 7, 10, 14, 35, and 70.

2.

| 12 | |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| ~~4~~ | ~~3~~ |

Mike can arrange the butterflies in 1 row of 12, 2 rows of 6, 3 rows of 4, 4 rows of 3, 6 rows of 2, or 12 rows of 1.

3. a. First, list the common factors of 24 and 36; then circle the common factors.

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

| 36 | |
|---|---|
| 1 | 36 |
| 2 | 18 |
| 3 | 12 |
| 4 | 9 |
| ~~5~~ | |
| 6 | 6 |

The common factors of 24 and 36 are 1, 2, 3, 4, 6, and 12.

b.

| 12 | |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| ~~4~~ | ~~3~~ |

| 18 | |
|---|---|
| 1 | 18 |
| 2 | 9 |
| 3 | 6 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | ~~3~~ |

The common factors of 12 and 18 are 1, 2, 3, and 6.

c.

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

| 75 | |
|---|---|
| 1 | 75 |
| ~~2~~ | |
| 3 | 25 |
| ~~4~~ | |
| 5 | 15 |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | |
| ~~10~~ | |
| ~~11~~ | |
| ~~12~~ | |
| ~~13~~ | |
| ~~14~~ | |
| ~~15~~ | ~~5~~ |

The common factors of 30 and 75 are 1, 3, 5, and 15.

**4. a.**

| 6 | |
|---|---|
| 1 | $\boxed{6}$ |
| 2 | 3 |
| ~~3~~ | ~~2~~ |

| 12 | |
|---|---|
| 1 | 12 |
| 2 | $\boxed{6}$ |
| 3 | 4 |
| ~~4~~ | ~~3~~ |

The greatest common factor is 6.

**b.**

| 30 | |
|---|---|
| 1 | 30 |
| 2 | $\boxed{15}$ |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

| 45 | |
|---|---|
| 1 | 45 |
| ~~2~~ | |
| 3 | $\boxed{15}$ |
| ~~4~~ | |
| 5 | 9 |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | ~~5~~ |

The greatest common factor is 15.

**c.**

| 42 | |
|---|---|
| 1 | 42 |
| 2 | 21 |
| 3 | $\boxed{14}$ |
| ~~4~~ | |
| ~~5~~ | |
| 6 | 7 |
| ~~7~~ | ~~6~~ |

| 56 | |
|---|---|
| 1 | 56 |
| 2 | 28 |
| ~~3~~ | |
| 4 | $\boxed{14}$ |
| ~~5~~ | |
| ~~6~~ | |
| 7 | 8 |
| ~~8~~ | ~~7~~ |

The greatest common factor is 14.

**5. a.**

| 54 | |
|---|---|
| 1 | 54 |
| 2 | 27 |
| 3 | 18 |
| ~~4~~ | |
| ~~5~~ | |
| 6 | 9 |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | ~~6~~ |

| 63 | |
|---|---|
| 1 | 63 |
| ~~2~~ | |
| 3 | 21 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | |
| 7 | 9 |
| ~~8~~ | |
| ~~9~~ | ~~7~~ |

| 81 | |
|---|---|
| 1 | 81 |
| ~~2~~ | |
| 3 | 27 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| 9 | 9 |

The greatest common factor is 9.

**b.**

| 8 | |
|---|---|
| 1 | 8 |
| 2 | 4 |
| ~~3~~ | |
| ~~4~~ | ~~2~~ |

| 16 | |
|---|---|
| 1 | 16 |
| 2 | 8 |
| ~~3~~ | |
| 4 | 4 |

| 20 | |
|---|---|
| 1 | 20 |
| 2 | 10 |
| ~~3~~ | |
| 4 | 5 |
| ~~5~~ | ~~4~~ |

The greatest common factor is 4.

c.

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

| 42 | |
|---|---|
| 1 | 42 |
| 2 | 21 |
| 3 | 14 |
| ~~4~~ | |
| ~~5~~ | |
| 6 | 7 |
| ~~7~~ | ~~6~~ |

| 48 | |
|---|---|
| 1 | 48 |
| 2 | 24 |
| 3 | 16 |
| 4 | 12 |
| ~~5~~ | |
| 6 | 8 |
| ~~7~~ | |
| ~~8~~ | ~~6~~ |

The greatest common factor is 6.

6.

| 28 | |
|---|---|
| 1 | 28 |
| 2 | 14 |
| ~~3~~ | |
| 4 | 7 |
| ~~5~~ | |
| ~~6~~ | |
| ~~7~~ | ~~4~~ |

| 72 | |
|---|---|
| 1 | 72 |
| 2 | 36 |
| 3 | 24 |
| 4 | 18 |
| ~~5~~ | |
| 6 | 12 |
| ~~7~~ | |
| 8 | 9 |
| ~~9~~ | ~~8~~ |

| 96 | |
|---|---|
| 1 | 96 |
| 2 | 48 |
| 3 | 32 |
| 4 | 24 |
| ~~5~~ | |
| 6 | 16 |
| ~~7~~ | |
| 8 | 12 |
| ~~9~~ | |
| ~~10~~ | |
| ~~11~~ | |
| ~~12~~ | ~~8~~ |

The greatest common factor is 4. So, the most that each jawbreaker could cost is 4¢.

## Now Try This

7. The number of members must be 1 greater than the least common multiple of 2, 3, 4, and 5. The LCM of 2, 3, 4, and 5 is 60. Therefore, the least number of members that could have been in the marching band is 61.

8. The LCM of 36 and 45 is 180. Since $180 \text{ min} = 3 \text{ h}$, the starting times of the videos will match up every 3 h. Thus, the starting times will match up at 8:45 A.M., 11:45 A.M., and 2:45 P.M.

## Looking Back

9. The greatest common factor of 36 and 81 is 9. Divide both the numerator and denominator by 9.

$$\frac{36}{81} \overset{\div 9}{=} \frac{4}{9}$$

## Section 1: Activity 4

1. **Note:** In this question, the factor trees may vary, but the resulting prime factorizations will be the same.

a.
$$\begin{array}{c} 38 \\ 2\times 19 \end{array}$$

So, $38 = 2\times 19$.

b.
$$\begin{array}{c} 42 \\ 2\times 21 \\ 2\times 3\times 7 \end{array} \quad \text{or} \quad \begin{array}{c} 42 \\ 6\times 7 \\ 2\times 3\times 7 \end{array}$$

So, $42 = 2\times 3\times 7$.

c.
$$\begin{array}{c} 45 \\ 3\times 15 \\ 3\times 3\times 5 \end{array} \quad \text{or} \quad \begin{array}{c} 45 \\ 5\times 9 \\ 5\times 3\times 3 \end{array}$$

So, $45 = 3\times 3\times 5$.

2. **Note:** In this question, the factor trees may vary, but the resulting prime factorizations will be the same.

**a.**

$24 \to 4 \times 6 \to 2 \times 2 \times 2 \times 3$

$36 \to 6 \times 6 \to 2 \times 3 \times 2 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$GCF = 2 \times 2 \times 3$

$= 12$

The greatest common factor of 24 and 36 is 12.

**b.**

$12 \to 3 \times 4 \to 3 \times 2 \times 2$

$18 \to 3 \times 6 \to 3 \times 2 \times 3$

$12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

$GCF = 2 \times 3$

$= 6$

The greatest common factor of 12 and 18 is 6.

**c.**

$20 \to 4 \times 5 \to 2 \times 2 \times 5$

$30 \to 5 \times 6 \to 5 \times 2 \times 3$

$75 \to 3 \times 25 \to 3 \times 5 \times 5$

$20 = 2 \times 2 \times 5$

$30 = 2 \times 3 \times 5$

$75 = 3 \times 5 \times 5$

The only complete column has 5s in it.

$GCF = 5$

The greatest common factor of 20, 30, and 75 is 5.

3. **a.**

$24 \to 4 \times 6 \to 2 \times 2 \times 2 \times 3$

$36 \to 6 \times 6 \to 2 \times 3 \times 2 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$LCM = 2 \times 2 \times 2 \times 3 \times 3$

$= 72$

The least common multiple of 24 and 36 is 72.

**b.**

$$\begin{array}{c} 12 \\ 3\times 4 \\ 3\times 2\times 2 \end{array} \qquad \begin{array}{c} 18 \\ 3\times 6 \\ 3\times 2\times 3 \end{array}$$

$$\begin{array}{lllllll} 12 = & 2 & \times 2 & \times 3 & \\ 18 = & 2 & & \times 3 & \times 3 \end{array}$$

$$\text{LCM} = 2\times 2\times 3\times 3$$
$$= 36$$

The least common multiple of 12 and 18 is 36.

**c.**

$$\begin{array}{c} 20 \\ 4\times 5 \\ 2\times 2\times 5 \end{array} \qquad \begin{array}{c} 30 \\ 5\times 6 \\ 5\times 2\times 3 \end{array} \qquad \begin{array}{c} 75 \\ 3\times 25 \\ 3\times 5\times 5 \end{array}$$

$$\begin{array}{llllll} 20 = & 2 & \times 2 & & \times 5 & \\ 30 = & 2 & & \times 3 & \times 5 & \\ 75 = & & & 3 & \times 5 & \times 5 \end{array}$$

$$\text{LCM} = 2\times 2\times 3\times 5\times 5$$
$$= 300$$

The least common multiple of 20, 30, and 75 is 300.

## Now Try This

**4.** One way to solve this problem is to use the guess, check, and revise method. This method is illustrated using the following table.

| Guess | Number of \$0.40 Fudge | Number of \$0.60 Fudge | Number of \$0.30 Fudge | Test |
|---|---|---|---|---|
| 1 | 4 | 4 | 7 | $4\times 0.40 = 1.60$<br>$4\times 0.60 = 2.40$<br>$7\times 0.30 = \underline{2.10}$<br>Total 6.10 |
| 2 | 5 | 5 | 5 | $5\times 0.40 = 2.00$<br>$5\times 0.60 = 3.00$<br>$5\times 0.30 = \underline{1.50}$<br>Total 6.50 |
| 3 | 6 | 6 | 3 | $6\times 0.40 = 2.40$<br>$6\times 0.60 = 3.60$<br>$3\times 0.30 = \underline{0.90}$<br>Total 6.90 |

The third guess gives the proper total. So, Samantha bought three pieces of the \$0.30 fudge.

## Looking Back

5. **Method 1**

$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \quad \times 7$$

Multiply one factor from each column to find the LCM.

$$\begin{aligned} LCM &= 2\times3\times5\times7 \\ &= 210 \end{aligned}$$

**Method 2**

List the multiples of 30 and 42.

30, 60, 90, 120, 150, 180, 210, 240, …
42, 84, 126, 168, 210, 252, …

The least common multiple of 30 and 42 is 210.

6. **Method 1**

$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \quad \times 7$$

Multiply the common factors to find the GCF.

$$\begin{aligned} GCF &= 2\times3 \\ &= 6 \end{aligned}$$

**Method 2**

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | $\boxed{6}$ |
| ~~6~~ | ~~5~~ |

| 42 | |
|---|---|
| 1 | 42 |
| 2 | 21 |
| 3 | 14 |
| ~~4~~ | |
| ~~5~~ | |
| $\boxed{6}$ | 7 |
| ~~7~~ | ~~6~~ |

The greatest common factor is 6.

## Section 2: Activity 1

1. Level $4 = +4$, Level $3 = +3$, Level $2 = +2$, Level $1 = +1$, Lobby $= 0$, Level $B1 = -1$, Level $B2 = -2$, and Level $B3 = -3$.

2. **a.** $-2$ **b.** $+5$

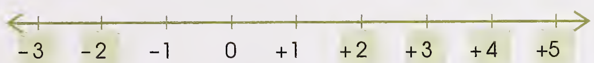
3. a. $+5000$ b. $-800$ c. $+20$ d. $-75$
e. $0$ f. $+100$ g. $-45$

4. a. $-36°C$, Inuvik
b. $13°C$, Calgary
c. Halifax was warmer, since $-6°C$ is warmer than $-7°C$.
d. Charlottetown was colder, since $-7°C$ is colder than $-5°C$.

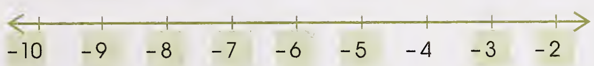
5. a. $|-3| = 3$ b. $|+7| = 7$ c. $|+5| = 5$ d. $|-4| = 4$

6. a. $+2$ b. $-8$ c. $-6$ d. $+3$

7. a.




b.



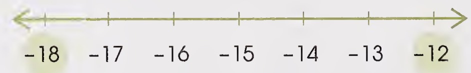

8. a. $6 < 16$ b. $-5 < -3$ c. $-2 > -20$
d. $-12 < 1$ e. $+4 < +5$ f. $0 > -3$

9. a. $-4,\ -3,\ 0,\ 8,\ 12$

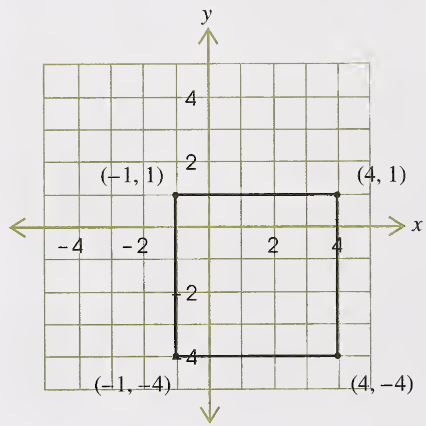
b. $-10,\ -8,\ -5,\ -4,\ 0,\ +4,\ +6,\ +11$

c. $-15,\ -6,\ -3,\ -1,\ +8,\ +11,\ +13$

10. No, Cyril's conclusion is not true; $-18 < -12$. Numbers increase as you move from left to right on the number line, and $-18$ is to the left of $-12$.

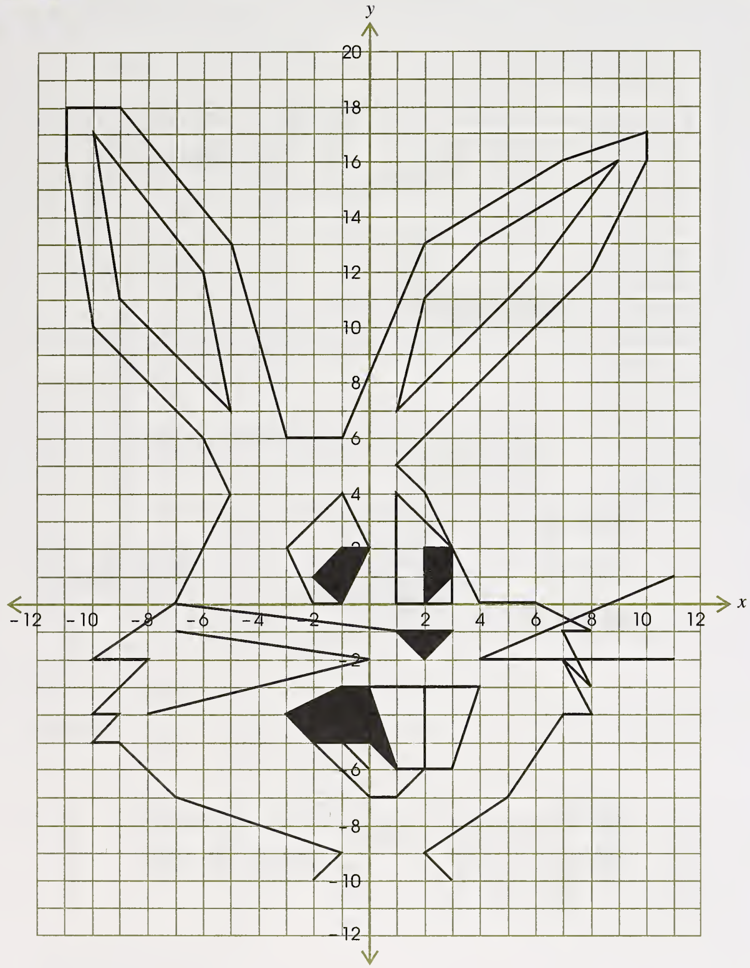



11.




A square is created.

12.



## Looking Back

**13.** You may have mentioned any situation that uses positive and negative numbers. Here are some examples:

- Integers are used in golf to show scores above and below par.
- Integers are used with the Celsius thermometer to show temperatures above the freezing point and below the freezing point.
- Integers are used to show business profits and losses.

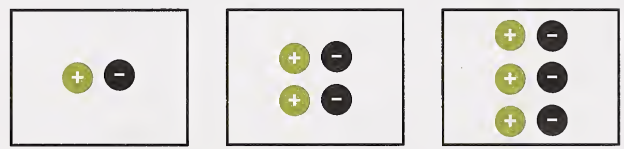
## Section 2: Activity 2

**1.** **a.** $-3$ **b.** $+4$ **c.** $0$

**d.** $+3$ **e.** $-4$ **f.** $+4$

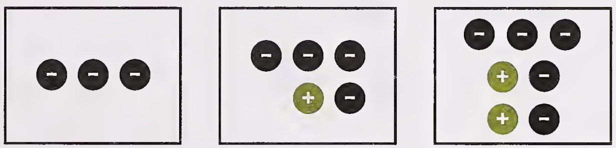
**2.** Answers will vary. Three sample answers are given for each integer.

**a.** To model 0, the number of positive and negative counters will always be equal.

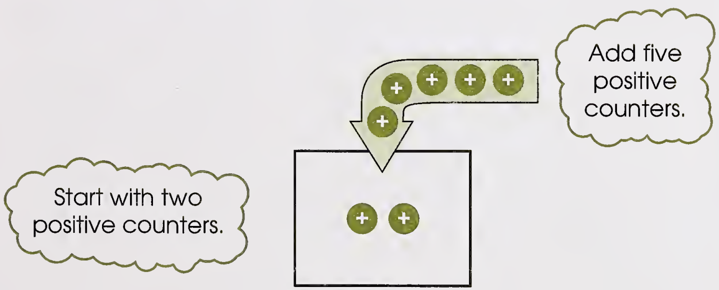


**b.** To model $+1$, you will always have one more positive counter than negative counters.

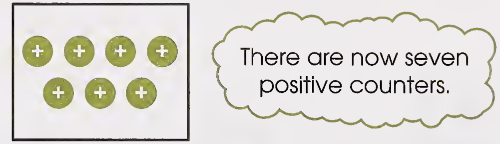
**c.** To model $-3$, you will always have three more negative counters than positive counters.



**3. a. Step 1:** Model the expression $(+2)+(+5)$.

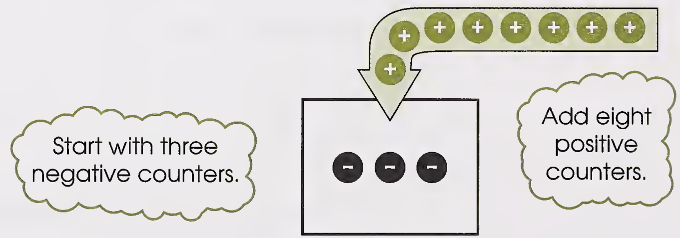



**Step 2:** Find the sum.

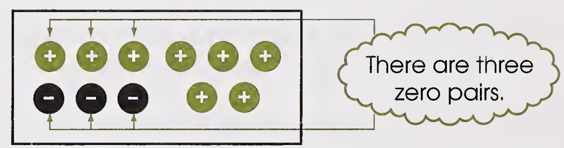



$\therefore\ (+2)+(+5)=+7$

**b. Step 1:** Model the expression $(-3)+(+8)$.



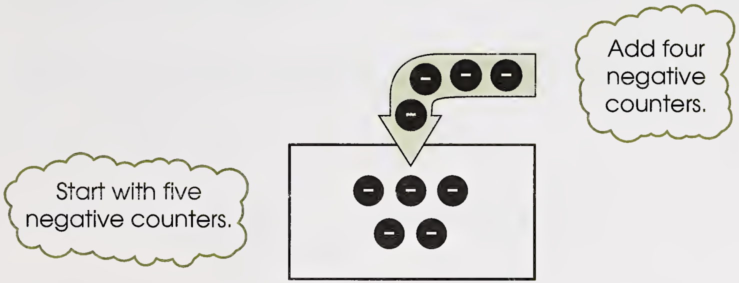

**Step 2:** Make as many zero pairs as possible.



**Step 3:** Find the sum.


$\therefore\ (-3)+(+8)=+5$

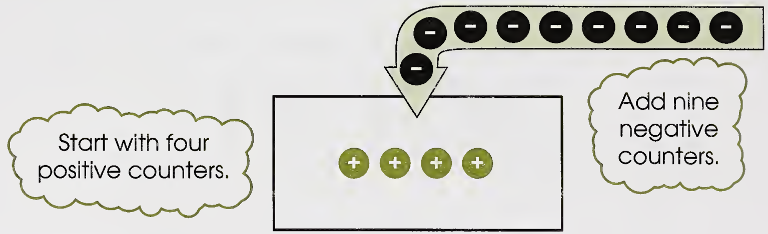
c. **Step 1:** Model the expression $(-5)+(-4)$.



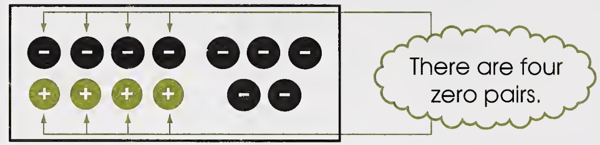

**Step 2:** Find the sum.


$\therefore\ (-5)+(-4)=-9$

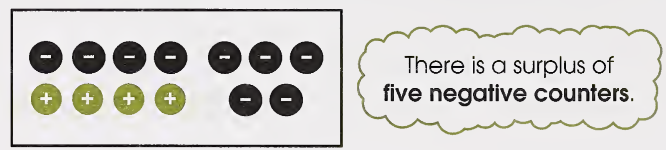
d. **Step 1:** Model the expression $(+4)+(-9)$.




**Step 2:** Make as many zero pairs as possible.




**Step 3:** Find the sum.




$\therefore\ (+4)+(-9)=-5$

4. a. $(-3)+(-5)=-8$

b. $(-1)+(+6)=+5$

c. $(-6)+(+8)=+2$

d. $\begin{array}{r}(+5)\\+(-5)\\\hline 0\end{array}$

e. $\begin{array}{r}(-5)\\+(+3)\\\hline -2\end{array}$

f. $\begin{array}{r}(+1)\\+(+4)\\\hline +5\end{array}$

g. $-7+5=-2$

h. $-4+(-3)=-7$

5. a. $(+45)+(-10)=+35$

The speed increased by 35 km/h altogether.

b. $(-400)+(+100)=-300$

The airplane descended 300 m altogether.

c. $(-3)+(-4)=-7$

The temperature fell 7°C altogether.

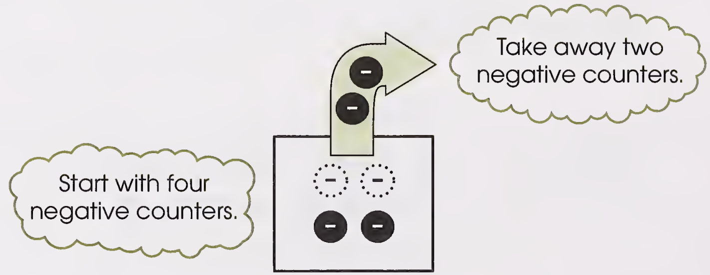
d. $(-50)+(-40)=-90$

Jasmine's account decreased \$90.

e. $(+420)+(-100)=+320$

Frank's account rose \$320 altogether.

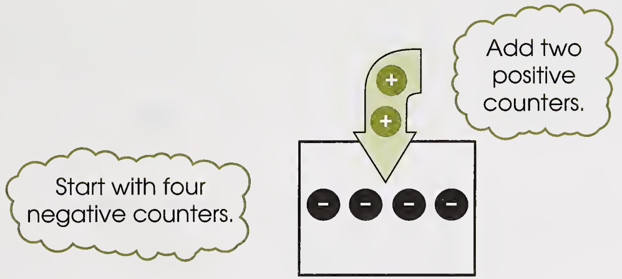
6. a. **Step 1:** Model the expression $(-4)-(-2)$. Start with four negative counters; then take away two negative counters.




**Step 2:** Find the difference. **Hint:** Are the remaining counters negative or positive? How many are there?

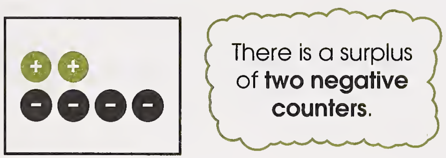

$\therefore\ (-4)-(-2)=-2$

b. **Step 1:** Model the expression $(-4)+(+2)$. Start with four negative counters; then add two positive counters.




**Step 2:** Because there is a combination of positive and negative counters, rearrange the counters making as many zero pairs as possible.


**Step 3:** Find the sum. **Hint:** Are the surplus counters positive or negative? How many are there?

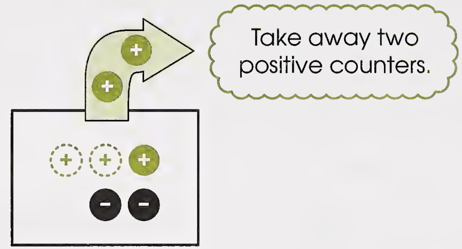



$\therefore\ (-4)+(+2)=-2$

c. The value of each expression is the same, $-2$.

7. a. **Step 1:** Model the expression $(+1)-(+2)$. Start with one positive counter; then take out two positive counters.

**Hint:** You will need to add sufficient zero pairs to do this. You can add as many zero pairs as you need, since their effect is the same as adding zero.

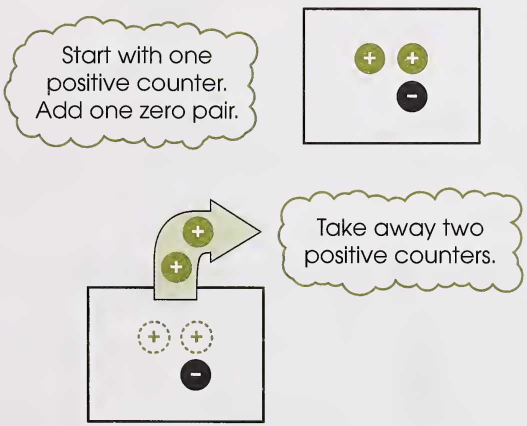




**Step 2:** Because there is a combination of positive and negative counters, rearrange the counters making as many zero pairs as possible.

**Step 3:** Find the difference. **Hint:** Are the surplus counters positive or negative? How many are there?


$\therefore\ (+1)-(+2)=-1$

**Note:** When finding the difference of $(+1)-(+2)$, you could have added just one zero pair.

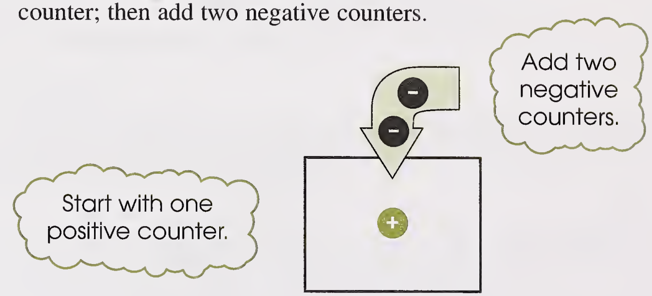



You are left with one negative counter.

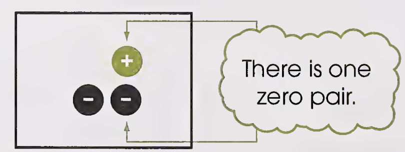
$\therefore\ (+1)-(+2)=-1$

**Remember:** You might have done this subtraction by adding any number of zero pairs.

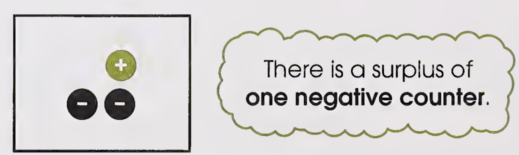
**b.** **Step 1:** Model the expression $(+1)+(-2)$. Start with one positive counter; then add two negative counters.




**Step 2:** Because there is a combination of positive and negative counters, rearrange the counters, making as many zero pairs as possible.




**Step 3:** Find the sum. **Hint:** Are the surplus counters positive or negative? How many are there?




$\therefore\ (+1)+(-2)=-1$

**c.** The value of each expression is the same, $-1$.

**8. a.** $(-7)-(-9)=-7+(+9)$
$=+2$

**b.** $(+4)-(+6)=(+4)+(-6)$
$=-2$

**c.** $(-3)-(+5)=(-3)+(-5)$
$=-8$

**d.** $\begin{array}{r}(+6)\\ -(-3)\\ \hline\end{array} \Rightarrow \begin{array}{r}(+6)\\ +(+3)\\ \hline +9\end{array}$

**e.** $\begin{array}{r}(-8)\\ -(-2)\\ \hline\end{array} \Rightarrow \begin{array}{r}(-8)\\ +(+2)\\ \hline -6\end{array}$

**f.** $\begin{array}{r}(+9)\\ -(+7)\\ \hline\end{array} \Rightarrow \begin{array}{r}+9\\ +(-7)\\ \hline +2\end{array}$

**9. a.** Final price – Beginning price = Difference

$$(+2)-(+5)=(+2)+(-5)$$
$$=-3$$

The share price fell $3.

**b.** Final location – Beginning location = Difference

$$(+3)-(-2)=(+3)+(+2)$$
$$=+5$$

The elevator rose 5 floors.

**c.** Final position – Beginning location = Difference

$$(-4)-(-3)=(-4)+(+3)$$
$$=-1$$

The submarine descended 1 km.

## Now Try This

**10.** You can use the guess, check, and revise strategy to solve this problem. You might have started with any other number as your first integer, but will end up with the only two integers that fit the condition of the question. The integers are $-6$ and $+1$.

| First Integer | 3 | 2 | 1 |
|---|---|---|---|
| Second Integer | −4 | −5 | −6 |

correct guess

**11.** You can use logic to solve this problem. The ship is floating on the water. As the tide rises, the ship rises; thus, the ladder will also rise. So, the ladder will be in the same position compared to the water level.

So, 5 m of the ladder will be submerged.

## Looking Back

**12. a.** To add integers with like signs, imagine the counters. When you added two sets of the same type of counters, you ended up with more of the same kind of counters. Therefore, when you add integers with like signs, you add the absolute values of the integers and use the sign of the integers.

Look at these examples:

$$(-5)+(-3)=-(5+3) \qquad (+7)+(+4)=+(7+4)$$
$$=-8 \qquad\qquad =+11$$

**b.** To add integers with unlike signs, imagine the counters. When you added two sets of counters that were unlike, you made as many zero pairs as possible and then determined the number and type of surplus counters remaining.

Therefore, when you add integers with unlike signs, you subtract the absolute values of the integers and use the sign of the integer with the greater absolute value.

Look at these examples:

$$\begin{aligned}(-8)+(+2)&=-(8-2)\\&=-6\end{aligned}\qquad\begin{aligned}(-5)+(+7)&=+(7-5)\\&=+2\end{aligned}$$

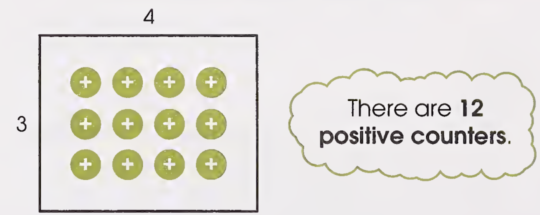
**c.** When you subtract integers, you can rewrite the subtraction as addition by adding the opposite. Then you can follow the addition methods for integers.

Look at these examples:

$$\begin{aligned}(-5)-(+2)&=(-5)+(-2)\\&=-7\end{aligned}\qquad\begin{aligned}(+2)-(-3)&=(+2)+(+3)\\&=+5\end{aligned}$$

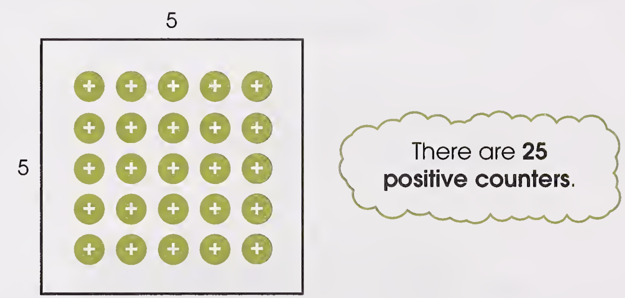
## Section 2: Activity 3

**1. a.**




$\therefore\ (+3)\times(+4)=+12$

**b.**




$\therefore\ (+5)\times(+5)=+25$

7. **a.** yes **b.** no **c.** yes **d.** yes **e.** no **f.** no

8. **a.** yes **b.** no **c.** yes **d.** no **e.** yes **f.** yes

9. **a.** no **b.** yes **c.** yes

10. **a.** no **b.** yes **c.** no

11. **a.** yes **b.** yes **c.** no

## Now Try This

12. You can solve the problem by finding and applying a pattern.

| Number of Students | Number of Games | Pattern |
|---|---|---|
| 1 | 0 | |
| 2 | 1 | +1 |
| 3 | 3 | +2 |
| 4 | 6 | +3 |
| 5 | 10 | +4 |
| 6 | 15 | +5 |
| 7 | 21 | +6 |
| 8 | 28 | +7 |

In the tournament, 28 games of chess will be played.

## Looking Back

13. To be divisible by 4, the last 2 digits must be divisible by 4. The number will be of the form 1 ___ ___ ___, where the last 2 digits are 04, 08, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, or 96.

One such example is 1560. You know this number is divisible by 4 because 60 is divisible by 4.

$$\begin{array}{r} 390 \\ 4\overline{)1560} \\ \underline{12}\phantom{0} \\ 36\phantom{0} \\ \underline{36}\phantom{0} \\ 0 \end{array}$$

Two factors of 1560 are 4 and 390.

## Section 1: Activity 3

1. **a.**

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

**b.**

| 32 | |
|---|---|
| 1 | 32 |
| 2 | 16 |
| ~~3~~ | |
| 4 | 8 |
| ~~5~~ | |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | ~~4~~ |

The factors of 32 are 1, 2, 4, 8, 16, and 32.

c.

| 45 | |
|---|---|
| 1 | 45 |
| ~~2~~ | |
| 3 | 15 |
| ~~4~~ | |
| 5 | 9 |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | ~~5~~ |

The factors of 45 are 1, 3, 5, 9, 15, and 45.

d.

| 70 | |
|---|---|
| 1 | 70 |
| 2 | 35 |
| ~~3~~ | |
| ~~4~~ | |
| 5 | 14 |
| ~~6~~ | |
| 7 | 10 |
| ~~8~~ | |
| ~~9~~ | |
| ~~10~~ | ~~7~~ |

The factors of 70 are 1, 2, 5, 7, 10, 14, 35, and 70.

2.

| 12 | |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| ~~4~~ | ~~3~~ |

Mike can arrange the butterflies in 1 row of 12, 2 rows of 6, 3 rows of 4, 4 rows of 3, 6 rows of 2, or 12 rows of 1.

3. a. First, list the common factors of 24 and 36; then circle the common factors.

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

| 36 | |
|---|---|
| 1 | 36 |
| 2 | 18 |
| 3 | 12 |
| 4 | 9 |
| ~~5~~ | |
| 6 | 6 |

The common factors of 24 and 36 are 1, 2, 3, 4, 6, and 12.

b.

| 12 | |
|---|---|
| 1 | 12 |
| 2 | 6 |
| 3 | 4 |
| ~~4~~ | ~~3~~ |

| 18 | |
|---|---|
| 1 | 18 |
| 2 | 9 |
| 3 | 6 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | ~~3~~ |

The common factors of 12 and 18 are 1, 2, 3, and 6.

c.

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

| 75 | |
|---|---|
| 1 | 75 |
| ~~2~~ | |
| 3 | 25 |
| ~~4~~ | |
| 5 | 15 |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | |
| ~~10~~ | |
| ~~11~~ | |
| ~~12~~ | |
| ~~13~~ | |
| ~~14~~ | |
| ~~15~~ | ~~5~~ |

The common factors of 30 and 75 are 1, 3, 5, and 15.

4. a.

| 6 | |
|---|---|
| 1 | $\boxed{6}$ |
| 2 | 3 |
| ~~3~~ | ~~2~~ |

| 12 | |
|---|---|
| 1 | 12 |
| 2 | $\boxed{6}$ |
| 3 | 4 |
| ~~4~~ | ~~3~~ |

The greatest common factor is 6.

b.

| 30 | |
|---|---|
| 1 | 30 |
| 2 | $\boxed{15}$ |
| 3 | 10 |
| ~~4~~ | |
| 5 | 6 |
| ~~6~~ | ~~5~~ |

| 45 | |
|---|---|
| 1 | 45 |
| ~~2~~ | |
| 3 | $\boxed{15}$ |
| ~~4~~ | |
| 5 | 9 |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | ~~5~~ |

The greatest common factor is 15.

c.

| 42 | |
|---|---|
| 1 | 42 |
| 2 | 21 |
| 3 | $\boxed{14}$ |
| ~~4~~ | |
| ~~5~~ | |
| 6 | 7 |
| ~~7~~ | ~~6~~ |

| 56 | |
|---|---|
| 1 | 56 |
| 2 | 28 |
| ~~3~~ | |
| 4 | $\boxed{14}$ |
| ~~5~~ | |
| ~~6~~ | |
| 7 | 8 |
| ~~8~~ | ~~7~~ |

The greatest common factor is 14.

5. a.

| 54 | |
|---|---|
| 1 | 54 |
| 2 | 27 |
| 3 | 18 |
| ~~4~~ | |
| ~~5~~ | |
| 6 | 9 |
| ~~7~~ | |
| ~~8~~ | |
| ~~9~~ | ~~6~~ |

| 63 | |
|---|---|
| 1 | 63 |
| ~~2~~ | |
| 3 | 21 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | |
| 7 | 9 |
| ~~8~~ | |
| ~~9~~ | ~~7~~ |

| 81 | |
|---|---|
| 1 | 81 |
| ~~2~~ | |
| 3 | 27 |
| ~~4~~ | |
| ~~5~~ | |
| ~~6~~ | |
| ~~7~~ | |
| ~~8~~ | |
| 9 | 9 |

The greatest common factor is 9.

b.

| 8 | |
|---|---|
| 1 | 8 |
| 2 | 4 |
| ~~3~~ | |
| ~~4~~ | ~~2~~ |

| 16 | |
|---|---|
| 1 | 16 |
| 2 | 8 |
| ~~3~~ | |
| 4 | 4 |

| 20 | |
|---|---|
| 1 | 20 |
| 2 | 10 |
| ~~3~~ | |
| 4 | 5 |
| ~~5~~ | ~~4~~ |

The greatest common factor is 4.

c.

| 24 | |
|---|---|
| 1 | 24 |
| 2 | 12 |
| 3 | 8 |
| 4 | 6 |
| ~~5~~ | |
| ~~6~~ | ~~4~~ |

| 42 | |
|---|---|
| 1 | 42 |
| 2 | 21 |
| 3 | 14 |
| ~~4~~ | |
| ~~5~~ | |
| 6 | 7 |
| ~~7~~ | ~~6~~ |

| 48 | |
|---|---|
| 1 | 48 |
| 2 | 24 |
| 3 | 16 |
| 4 | 12 |
| ~~5~~ | |
| 6 | 8 |
| ~~7~~ | |
| ~~8~~ | ~~6~~ |

The greatest common factor is 6.

6.

| 28 | |
|---|---|
| 1 | 28 |
| 2 | 14 |
| ~~3~~ | |
| 4 | 7 |
| ~~5~~ | |
| ~~6~~ | |
| ~~7~~ | ~~4~~ |

| 72 | |
|---|---|
| 1 | 72 |
| 2 | 36 |
| 3 | 24 |
| 4 | 18 |
| ~~5~~ | |
| 6 | 12 |
| ~~7~~ | |
| 8 | 9 |
| ~~9~~ | ~~8~~ |

| 96 | |
|---|---|
| 1 | 96 |
| 2 | 48 |
| 3 | 32 |
| 4 | 24 |
| ~~5~~ | |
| 6 | 16 |
| ~~7~~ | |
| 8 | 12 |
| ~~9~~ | |
| ~~10~~ | |
| ~~11~~ | |
| ~~12~~ | ~~8~~ |

The greatest common factor is 4. So, the most that each jawbreaker could cost is 4¢.

## Now Try This

7. The number of members must be 1 greater than the least common multiple of 2, 3, 4, and 5. The LCM of 2, 3, 4, and 5 is 60. Therefore, the least number of members that could have been in the marching band is 61.

8. The LCM of 36 and 45 is 180. Since $180\ \text{min} = 3\ \text{h}$, the starting times of the videos will match up every 3 h. Thus, the starting times will match up at 8:45 A.M., 11:45 A.M., and 2:45 P.M.

## Looking Back

9. The greatest common factor of 36 and 81 is 9. Divide both the numerator and denominator by 9.

$$\frac{36}{81} \overset{\div 9}{=} \frac{4}{9}$$

## Section 1: Activity 4

1. **Note:** In this question, the factor trees may vary, but the resulting prime factorizations will be the same.

**a.** 38

$2 \times 19$

So, $38 = 2 \times 19$.

**b.** 42

$2 \times 21$

$2 \times 3 \times 7$

or

42

$6 \times 7$

$2 \times 3 \times 7$

So, $42 = 2 \times 3 \times 7$.

**c.** 45

$3 \times 15$

$3 \times 3 \times 5$

or

45

$5 \times 9$

$5 \times 3 \times 3$

So, $45 = 3 \times 3 \times 5$.

2. **Note:** In this question, the factor trees may vary, but the resulting prime factorizations will be the same.

**a.**

24
$4 \times 6$
$2 \times 2 \times 2 \times 3$

36
$6 \times 6$
$2 \times 3 \times 2 \times 3$

$$\begin{array}{llllll} 24 = & 2 \times & 2 & \times 2 \times & 3 & \\ 36 = & 2 \times & 2 & \times & 3 & \times 3 \end{array}$$

$$\begin{aligned} GCF &= 2 \times 2 \times 3 \\ &= 12 \end{aligned}$$

The greatest common factor of 24 and 36 is 12.

**b.**

12
$3 \times 4$
$3 \times 2 \times 2$

18
$3 \times 6$
$3 \times 2 \times 3$

$$\begin{array}{lllll} 12 = & 2 & \times 2 \times & 3 & \\ 18 = & 2 & \times & 3 & \times 3 \end{array}$$

$$\begin{aligned} GCF &= 2 \times 3 \\ &= 6 \end{aligned}$$

The greatest common factor of 12 and 18 is 6.

**c.**

20
$4 \times 5$
$2 \times 2 \times 5$

30
$5 \times 6$
$5 \times 2 \times 3$

75
$3 \times 25$
$3 \times 5 \times 5$

$$\begin{array}{lllll} 20 = 2 \times 2 & & \times & 5 & \\ 30 = 2 & \times 3 & \times & 5 & \\ 75 = & & 3 \times & 5 & \times 5 \end{array}$$

The only complete column has 5s in it.

$GCF = 5$

The greatest common factor of 20, 30, and 75 is 5.

3. **a.**

24
$4 \times 6$
$2 \times 2 \times 2 \times 3$

36
$6 \times 6$
$2 \times 3 \times 2 \times 3$

$$\begin{array}{llllllll} 24 = & 2 \times & 2 \times & 2 & \times & 3 & & \\ 36 = & 2 \times & 2 & & \times & 3 & \times & 3 \end{array}$$

$$\begin{aligned} LCM &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= 72 \end{aligned}$$

The least common multiple of 24 and 36 is 72.

**b.**

$$\begin{array}{c} 12 \\ 3 \times 4 \\ 3 \times 2 \times 2 \end{array} \qquad \begin{array}{c} 18 \\ 3 \times 6 \\ 3 \times 2 \times 3 \end{array}$$

$$\begin{array}{rccccccc} 12 = & 2 & \times & 2 & \times & 3 & & \\ 18 = & 2 & & & \times & 3 & \times & 3 \end{array}$$

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 3 \times 3 \\ &= 36 \end{aligned}$$

The least common multiple of 12 and 18 is 36.

**c.**

$$\begin{array}{c} 20 \\ 4 \times 5 \\ 2 \times 2 \times 5 \end{array} \qquad \begin{array}{c} 30 \\ 5 \times 6 \\ 5 \times 2 \times 3 \end{array} \qquad \begin{array}{c} 75 \\ 3 \times 25 \\ 3 \times 5 \times 5 \end{array}$$

$$\begin{array}{rccccccccc} 20 = & 2 & \times & 2 & & & \times & 5 & & \\ 30 = & 2 & & & \times & 3 & \times & 5 & & \\ 75 = & & & & & 3 & \times & 5 & \times & 5 \end{array}$$

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 3 \times 5 \times 5 \\ &= 300 \end{aligned}$$

The least common multiple of 20, 30, and 75 is 300.

## Now Try This

**4.** One way to solve this problem is to use the guess, check, and revise method. This method is illustrated using the following table.

| Guess | Number of \$0.40 Fudge | Number of \$0.60 Fudge | Number of \$0.30 Fudge | Test |
|---|---|---|---|---|
| 1 | 4 | 4 | 7 | $4 \times 0.40 = 1.60$<br>$4 \times 0.60 = 2.40$<br>$7 \times 0.30 = \underline{2.10}$<br>Total 6.10 |
| 2 | 5 | 5 | 5 | $5 \times 0.40 = 2.00$<br>$5 \times 0.60 = 3.00$<br>$5 \times 0.30 = \underline{1.50}$<br>Total 6.50 |
| 3 | 6 | 6 | 3 | $6 \times 0.40 = 2.40$<br>$6 \times 0.60 = 3.60$<br>$3 \times 0.30 = \underline{0.90}$<br>Total 6.90 |

The third guess gives the proper total. So, Samantha bought three pieces of the \$0.30 fudge.

## Looking Back

5. **Method 1**

$$\begin{array}{l} 30 = 2 \times 3 \times 5 \\ 42 = 2 \times 3 \quad\;\; \times 7 \end{array}$$

Multiply one factor from each column to find the LCM.

$$\begin{aligned} \text{LCM} &= 2\times3\times5\times7 \\ &= 210 \end{aligned}$$

**Method 2**

List the multiples of 30 and 42.

30, 60, 90, 120, 150, 180, 210, 240, …
42, 84, 126, 168, 210, 252, …

The least common multiple of 30 and 42 is 210.

6. **Method 1**

$$\begin{array}{l} 30 = 2 \times 3 \times 5 \\ 42 = 2 \times 3 \quad\;\; \times 7 \end{array}$$

Multiply the common factors to find the GCF.

$$\begin{aligned} \text{GCF} &= 2\times3 \\ &= 6 \end{aligned}$$

**Method 2**

| 30 | |
|---|---|
| 1 | 30 |
| 2 | 15 |
| 3 | 10 |
| $\cancel{4}$ | |
| 5 | $\boxed{6}$ |
| $\cancel{6}$ | $\cancel{5}$ |

| 42 | |
|---|---|
| 1 | 42 |
| 2 | 21 |
| 3 | 14 |
| $\cancel{4}$ | |
| $\cancel{5}$ | |
| $\boxed{6}$ | 7 |
| $\cancel{7}$ | $\cancel{6}$ |

The greatest common factor is 6.

## Section 2: Activity 1

1. Level $4 = +4$, Level $3 = +3$, Level $2 = +2$, Level $1 = +1$, Lobby $= 0$, Level B1 $= -1$, Level B2 $= -2$, and Level B3 $= -3$.

2. a. $-2$ b. $+5$

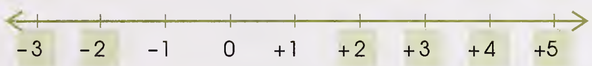
3. a. $+5000$  b. $-800$  c. $+20$  d. $-75$
   e. $0$  f. $+100$  g. $-45$

4. a. $-36°C$, Inuvik
   b. $13°C$, Calgary
   c. Halifax was warmer, since $-6°C$ is warmer than $-7°C$.
   d. Charlottetown was colder, since $-7°C$ is colder than $-5°C$.

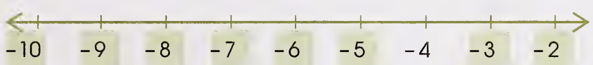
5. a. $|-3| = 3$  b. $|+7| = 7$  c. $|+5| = 5$  d. $|-4| = 4$

6. a. $+2$  b. $-8$  c. $-6$  d. $+3$

7. a.



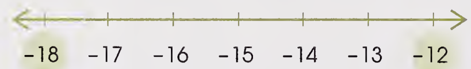

b.



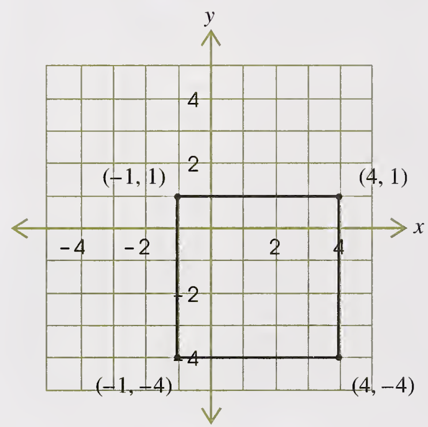

8. a. $6 < 16$  b. $-5 < -3$  c. $-2 > -20$
   d. $-12 < 1$  e. $+4 < +5$  f. $0 > -3$

9. a. $-4, -3, 0, 8, 12$
   b. $-10, -8, -5, -4, 0, +4, +6, +11$
   c. $-15, -6, -3, -1, +8, +11, +13$

10. No, Cyril's conclusion is not true; $-18 < -12$. Numbers increase as you move from left to right on the number line, and $-18$ is to the left of $-12$.




11.




A square is created.

13. The exponents are opposites: 2 is the opposite of $-2$, and 1 is the opposite of $-1$.

14. a. $4^{-3}=\frac{1}{4^3}$

b. $(-7)^{-4}=\frac{1}{(-7)^4}$

c. $0.6^{-5}=\frac{1}{0.6^5}$

## Now Try This

15. The number of loonies on a square on the board is given by a power of 2.

$2^0=1$ ⟵ first square
$2^1=2$ ⟵ second square
$2^2=4$ ⟵ third square
$2^3=8$ ⟵ fourth square
⋮
$2^{19}=524\ 288$ ⟵ twentieth square
$2^{20}=1\ 048\ 576$ ⟵ twenty-first square

Therefore, the first square to have over $1 000 000 on it is the twenty-first square.

16. The smallest number for which twice the number is smaller than the number squared is 3. This can be shown using the following pattern:

$1\times2=2$ and $1^2=1$
$2\times2=4$ and $2^2=4$
$3\times2=6$ and $3^2=9$
$4\times2=8$ and $4^2=16$

## Looking Back

17. If you were **multiplying** 2 by itself, a 4-digit number would result much quicker! In fact, the minimum number of times 2 has to be multiplied by itself to obtain a 4-digit number is **10** and the maximum number of times is **13**.

## Section 3: Activity 2

1.

| Expression | Factored Form | Power Form |
|---|---|---|
| $3^4\times3^3$ | $(3\times3\times3\times3)\times(3\times3\times3)$ $=3\times3\times3\times3\times3\times3\times3$ | $3^7$ |
| $4^2\times4^3$ | $(4\times4)\times(4\times4\times4)$ $=4\times4\times4\times4\times4$ | $4^5$ |
| $5\times5^3$ | $5\times(5\times5\times5)=5\times5\times5\times5$ | $5^4$ |
| $2^2\times2^3\times2^4$ | $(2\times2)\times(2\times2\times2)\times(2\times2\times2\times2)$ $=2\times2\times2\times2\times2\times2$ $\times2\times2\times2$ | $2^9$ |

2. You can obtain the answers for the multiplications in question 1 by adding the exponents. For example, $4^2 \times 4^3 = 4^{2+3} = 4^5$.

3. The shortcut does not work for $3^2 \times 4^5$ because the bases are different. You can only add the exponents when multiplying powers with identical bases.

4. a. $2^3 \times 2^7 = 2^{3+7}$
$= 2^{10}$

b. $(-5)^8 \times (-5)^{10} = (-5)^{8+10}$
$= (-5)^{18}$

c. This cannot be written as a single power because the bases are not the same.

d. $8^3 \times 8^4 \times 8^5 = 8^{3+4+5}$
$= 8^{12}$

e. $8^2 \times 8^{-3} = 8^{2+(-3)}$
$= 8^{-1}$

f. $2^{-3} \times 2^{-2} = 2^{-3+(-2)}$
$= 2^{-5}$

5. a. $10^{15} \times 10^4 = 10^{15+4}$
$= 10^{19}$

The electron will make $10^{19}$ orbits.

b. $10^{15} \times 10^{11} = 10^{15+11}$
$= 10^{26}$

The electron will make $10^{26}$ orbits.

6. $10^5 \times 10^4 = 10^{5+4}$
$= 10^9$

The signal was boosted by a factor of $10^9$ after passing through both amplifiers.

7.

| Expression | Factored Form | Power Form |
|---|---|---|
| $5^6 \div 5^2$ | $(5\times5\times5\times5\times5\times5)\div(5\times5)$ <br> $= \dfrac{\cancel{5}^{1}\times\cancel{5}^{1}\times5\times5\times5\times5}{\cancel{5}_{1}\times\cancel{5}_{1}}$ <br> $= 5\times5\times5\times5$ | $5^4$ |

| | | |
|---|---|---|
| $2^5 \div 2^3$ | $(2\times2\times2\times2\times2)\div(2\times2\times2)$ $=\frac{\cancel{2}^1\times\cancel{2}^1\times\cancel{2}^1\times2\times2}{\cancel{2}_1\times\cancel{2}_1\times\cancel{2}_1}$ $=2\times2$ | $2^2$ |
| $4^6 \div 4^5$ | $(4\times4\times4\times4\times4\times4)$ $\div(4\times4\times4\times4\times4)$ $=\frac{\cancel{4}^1\times\cancel{4}^1\times\cancel{4}^1\times\cancel{4}^1\times\cancel{4}^1\times4}{\cancel{4}_1\times\cancel{4}_1\times\cancel{4}_1\times\cancel{4}_1\times\cancel{4}_1}$ $=4$ | $4^1$ |
| $3^7 \div 3^4$ | $(3\times3\times3\times3\times3\times3\times3)$ $\div(3\times3\times3\times3)$ $=\frac{\cancel{3}^1\times\cancel{3}^1\times\cancel{3}^1\times\cancel{3}^1\times3\times3\times3}{\cancel{3}_1\times\cancel{3}_1\times\cancel{3}_1\times\cancel{3}_1}$ $=3\times3\times3$ | $3^3$ |
| $5^5 \div 5^1$ | $(5\times5\times5\times5\times5)\div(5)$ $=\frac{\cancel{5}^1\times5\times5\times5\times5}{\cancel{5}_1}$ $=5\times5\times5\times5$ | $5^4$ |

**8.** You can obtain the answers to the divisions in question 7 by subtracting the exponents. For example, $2^5 \div 2^3 = 2^{5-3} = 2^2$.

**9.** The rule does not work for $5^6 \div 2^3$ because the bases of the powers being divided are not the same.

**10.** **a.** $3^6 \div 3^2 = 3^{6-2}$
$= 3^4$

**b.** $(-5)^7 \div (-5)^3 = (-5)^{7-3}$
$= (-5)^4$

**c.** $\frac{5^7}{5^7} = 5^{7-7}$
$= 5^0$ or 1

**d.** This cannot be simplified because the bases are not the same.

**e.** $(-3)^2 \div (-3)^5 = (-3)^{2-5}$
$= (-3)^{-3}$

**f.** $4^{-2} \div 4^{-4} = 4^{-2-(-4)}$
$= 4^{-2+4}$
$= 4^2$

## Now Try This

**11.** **a.** $\frac{8^2 \times 8^3}{8^4} = 8^{2+3-4}$
$= 8^1$

**b.** $\frac{9^2 \times 9^3 \times 9^4}{9 \times 9^5} = \frac{9^{2+3+4}}{9^{1+5}}$
$= \frac{9^9}{9^6}$
$= 9^{9-6}$
$= 9^3$

c. $\left(4^2 \times 4^6 \times 4\right) \div \left(4^5 \times 4 \times 4\right) = \left(4^{2+6+1}\right) \div \left(4^{5+1+1}\right)$

$$= 4^9 \div 4^7$$
$$= 4^{9-7}$$
$$= 4^2$$

12. a. $p \times p \times p \times p \times p \times p \times p = p^7$

b. $m^2 \times m^4 = m^6$

c. $n \times n^2 \times n^3 = n^6$

d. $p^8 \div p^3 = p^5$

e. $\dfrac{a^6}{a^3} = a^3$

f. $\dfrac{a^3 b^6}{ab^2} = a^2 b^4$

## Looking Back

13. To multiply two expressions with the same base, add the exponents.

$$3^5 \times 3^4 = 3^{5+4}$$
$$= 3^9$$

To divide two expressions with the same base, subtract the exponents.

$$8^7 \div 8^3 = 8^{7-3}$$
$$= 8^4$$

14. You cannot write the product $3^5 \times 5^3$ as a single power because the bases are different.

## Section 3: Activity 3

1.

| Expression | Factored Form | Power Form |
|---|---|---|
| $\left(5^3\right)^2$ | $5^3 \times 5^3 = (5\times5\times5)\times(5\times5\times5)$ $= 5\times5\times5\times5\times5\times5$ | $5^6$ |
| $\left(7^2\right)^3$ | $7^2 \times 7^2 \times 7^2$ $= (7\times7)\times(7\times7)\times(7\times7)$ $= 7\times7\times7\times7\times7\times7$ | $7^6$ |
| $\left(3^4\right)^2$ | $3^4 \times 3^4$ $= (3\times3\times3\times3)$ $\times(3\times3\times3\times3)$ $= 3\times3\times3\times3\times3\times3\times3\times3$ | $3^8$ |
| $\left(2^3\right)^4$ | $2^3 \times 2^3 \times 2^3 \times 2^3$ $= (2\times2\times2)\times(2\times2\times2)$ $\times(2\times2\times2)\times(2\times2\times2)$ $= 2\times2\times2\times2\times2\times2$ $\times2\times2\times2\times2\times2\times2$ | $2^{12}$ |

2. You can obtain the answers in question 1 by multiplying the exponents.

For example, $\left(7^2\right)^3 = 7^{2\times3} = 7^6$.

3. a. $\left(3^5\right)^4 = 3^{5\times4}$
$= 3^{20}$

b. $\left[(-2)^3\right]^7 = (-2)^{3\times7}$
$= (-2)^{21}$

c. $\left(0.2^5\right)^3 = 0.2^{5\times3}$
$= 0.2^{15}$

d. $\left[(-0.1)^2\right]^6 = (-0.1)^{2\times6}$
$= (-0.1)^{12}$

e. $\left(4^2\right)^{-1} = 4^{2\times(-1)}$
$= 4^{-2}$

f. $\left(10^{-2}\right)^{-3} = 10^{(-2)\times(-3)}$
$= 10^6$

4. a. $\left(10^{10}\right)^{10} = 10^{10\times10}$
$= 10^{100}$

b. The exponent for a googolplex would be 1 followed by 100 zeros.

5. $\left(3^4\right)^2 = (3\times3\times3\times3)^2$
$= (3\times3\times3\times3)\times(3\times3\times3\times3)$ ← There are eight factors of 3.

$\left(3^2\right)^4 = (3\times3)^4$
$= (3\times3)\times(3\times3)\times(3\times3)\times(3\times3)$ ← There are eight factors of 3.

Since $\left(3^4\right)^2$ and $\left(3^2\right)^4$ have the same number of factors of 3, they are equivalent.

6.

| Expression | Factored Form | Power Form |
|---|---|---|
| $\left(6^2\times4^3\right)^3$ | $\left(6^2\times4^3\right)\times\left(6^2\times4^3\right)\times\left(6^2\times4^3\right)$ $=6^2\times6^2\times6^2\times4^3\times4^3\times4^3$ | $6^6\times4^9$ |
| $\left(5^4\times2^7\right)^2$ | $\left(5^4\times2^7\right)\times\left(5^4\times2^7\right)$ $=5^4\times5^4\times2^7\times2^7$ | $5^8\times2^{14}$ |
| $\left(8\times7^4\right)^3$ | $\left(8\times7^4\right)\times\left(8\times7^4\right)\times\left(8\times7^4\right)$ $=8\times8\times8\times7^4\times7^4\times7^4$ | $8^3\times7^{12}$ |
| $\left(3^5\times4^6\right)^2$ | $\left(3^5\times4^6\right)\times\left(3^5\times4^6\right)$ $=3^5\times3^5\times4^6\times4^6$ | $3^{10}\times4^{12}$ |

7. To simplify the power of a product, you can multiply each of the exponents inside the brackets by the exponent outside the brackets. For example, $\left(5^4\times2^7\right)^2 = 5^{4\times2}\times2^{7\times2} = 5^8\times2^{14}$.

8. a. $\left(8^7\times6^2\right)^4 = 8^{7\times4}\times6^{2\times4}$
$= 8^{28}\times6^8$

b. $\left[(-3)^2\times(-4)^7\right]^3 = (-3)^{2\times3}\times(-4)^{7\times3}$
$= (-3)^6\times(-4)^{21}$

c. $\left(7\times4^2\right)^5 = 7^{1\times5}\times4^{2\times5}$

$= 7^5\times4^{10}$

d. $\left(0.5^4\times1.4^3\right)^{-2} = 0.5^{4\times-2}\times1.4^{3\times-2}$

$= 0.5^{-8}\times1.4^{-6}$

9.

| Expression | Factored Form | Power Form |
|---|---|---|
| $\left(\frac{8^2}{4^3}\right)^3$ | $\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)\times\left(\frac{8^2}{4^3}\right)$ $=\frac{8^2\times8^2\times8^2}{4^3\times4^3\times4^3}$ | $\frac{8^6}{4^9}$ |
| $\left(\frac{5^4}{3^2}\right)^2$ | $\left(\frac{5^4}{3^2}\right)\times\left(\frac{5^4}{3^2}\right)$ $=\frac{5^4\times5^4}{3^2\times3^2}$ | $\frac{5^8}{3^4}$ |
| $\left(\frac{4^7}{2^5}\right)^3$ | $\left(\frac{4^7}{2^5}\right)\times\left(\frac{4^7}{2^5}\right)\times\left(\frac{4^7}{2^5}\right)$ $=\frac{4^7\times4^7\times4^7}{2^5\times2^5\times2^5}$ | $\frac{4^{21}}{2^{15}}=\frac{\left(2^2\right)^{21}}{2^{15}}$ $=\frac{2^{42}}{2^{15}}$ $=2^{42-15}$ $=2^{27}$ |

10. To simplify the power of a quotient, multiply each of the exponents inside the brackets by the exponent outside the brackets.

e.g., $\left(\frac{5^4}{3^2}\right)^2 = \frac{5^{4\times2}}{3^{2\times2}}$

$= \frac{5^8}{3^4}$

11. a. $\left(\frac{8^4}{5^3}\right)^3 = \frac{8^{4\times3}}{5^{3\times3}}$

$= \frac{8^{12}}{5^9}$

b. $\left(\frac{7^4}{3^5}\right)^6 = \frac{7^{4\times6}}{3^{5\times6}}$

$= \frac{7^{24}}{3^{30}}$

c. $\left(\frac{4^7}{3^4}\right)^{-3} = \frac{4^{7\times(-3)}}{3^{4\times(-3)}}$

$= \frac{4^{-21}}{3^{-12}}$

d. $\left[\frac{(-5)^{-2}}{(-2)^{-6}}\right]^{-2} = \frac{(-5)^{(-2)\times(-2)}}{(-2)^{(-6)\times(-2)}}$

$= \frac{(-5)^4}{(-2)^{12}}$

**12.** **a.** $\left(a^2\right)^3 = a^{2\times3}$
$= a^6$

**b.** $\left(y^3\right)^4 = y^{3\times4}$
$= y^{12}$

**c.** $\left(m^4 \times n^3\right)^2 = m^{4\times2} \times n^{3\times2}$
$= m^8 \times n^6$

**d.** $\left(a^2 b^2\right)^3 = a^{2\times3} b^{2\times3}$
$= a^6 b^6$

**e.** $\left(\dfrac{c^2}{b^3}\right)^2 = \dfrac{c^{2\times2}}{b^{3\times2}}$
$= \dfrac{c^4}{b^6}$

**f.** $\left(a^4 \div b^2\right)^3 = a^{4\times3} \div b^{2\times3}$
$= a^{12} \div b^6$

## Now Try This

**13.** **a.**

| | | Pattern |
|---|---|---|
| $1^3 = 1$ | $1^2$ | |
| | | base increased by 2 |
| $1^3 + 2^3 = 9$ | $3^2$ | |
| | | base increased by 3 |
| $1^3 + 2^3 + 3^3 = \boxed{36}$ | $6^2$ | |
| | | base increased by 4 |
| $1^3 + 2^3 + 3^3 + 4^3 = \boxed{100}$ | $10^2$ | |
| | | base increased by 5 |
| $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = \boxed{225}$ | $15^2$ | |

**b.** According to the pattern in the answer to question 13.a., the sum of the first six cubic numbers would be $21^2 = 441$.

## Looking Back

**14.** Your examples may vary, but should follow the pattern of these examples:

- power of a power

$$\left(6^3\right)^2 = (6\times6\times6)(6\times6\times6) = 6^6 \quad \text{or} \quad \left(6^3\right)^2 = 6^{3\times2} = 6^6$$

- power of a product

$$\left(5^2 \times 3^4\right)^3 = \left(5^2 \times 3^4\right)\left(5^2 \times 3^4\right)\left(5^2 \times 3^4\right)$$
$$= 5^2 \times 5^2 \times 5^2 \times 3^4 \times 3^4 \times 3^4$$
$$= 5^6 \times 3^{12}$$

or

$$\left(5^2 \times 3^4\right)^3 = 5^{2\times3} \times 3^{4\times3}$$
$$= 5^6 \times 3^{12}$$

- power of a quotient

$$\left(\frac{7^3}{5^2}\right)^2 = \left(\frac{7\times7\times7}{5\times5}\right)\left(\frac{7\times7\times7}{5\times5}\right) = \frac{7^6}{5^4} \quad \text{or} \quad \left(\frac{7^3}{5^2}\right)^2 = \frac{7^{3\times2}}{5^{2\times2}} = \frac{7^6}{5^4}$$

## Section 3: Activity 4

1. a. $$\begin{aligned}3^3-3\times2&=27-3\times2\\&=27-6\\&=21\end{aligned}$$

b. $$\begin{aligned}2^3-2^2&=8-4\\&=4\end{aligned}$$

c. $$\begin{aligned}&2^2+3^2+4^2-(2+3)^2\\&=2^2+3^2+4^2-5^2\\&=4+9+16-25\\&=4\end{aligned}$$

d. $$\begin{aligned}(-8)^2-5^2&=64-25\\&=39\end{aligned}$$

e. $$\begin{aligned}\frac{4^3}{4^2\times2}&=\frac{64}{16\times2}\\&=\frac{64}{32}\\&=2\end{aligned}$$

f. $$\begin{aligned}\frac{9^2-3\times6}{12-(4+1)}&=\frac{9^2-3\times6}{12-5}\\&=\frac{81-3\times6}{12-5}\\&=\frac{81-18}{12-5}\\&=\frac{63}{7}\\&=9\end{aligned}$$

g. $$\begin{aligned}\frac{(12-6)\times(12+6)}{(3+3)^2}&=\frac{6\times18}{6^2}\\&=\frac{\overset{1}{\cancel{6}}\times\overset{3}{\cancel{18}}}{\underset{1}{\cancel{36}}}\\&=3\end{aligned}$$

h. $$\begin{aligned}\frac{3^2\times4+3\times2}{16\div4+8+2}&=\frac{9\times4+3\times2}{16\div4+8+2}\\&=\frac{36+6}{4+8+2}\\&=\frac{42}{14}\\&=3\end{aligned}$$

2. a. $$\begin{aligned}-2\left[(-2)^2+(-3)^2+(-4)^2\right]&=-2(4+9+16)\\&=-2(29)\\&=-58\end{aligned}$$

b. $$\begin{aligned}100-\left[9^2-(13+12)\right]&=100-\left(9^2-25\right)\\&=100-(81-25)\\&=100-56\\&=44\end{aligned}$$

c. $$\begin{aligned}\left[(-8)^2-5\right]^2&=(64-5)^2\\&=59^2\\&=3481\end{aligned}$$

d. $$\begin{aligned}\left[\frac{(-3)^2+(-7)}{-2}\right]^3&=\left[\frac{9+(-7)}{-2}\right]^3\\&=\left(\frac{2}{-2}\right)^3\\&=(-1)^3\\&=-1\end{aligned}$$

3. a. 3 $x^y$ 3 − 3 × 2 =

21.

b. 2 $x^y$ 3 − 2 $x^2$ =

4.

c. 2 $x^2$ − 3 $x^2$ + 4 $x^2$ − ( 2 − 3 ) $x^2$ =

4.

d. ( 8 +/_ ) $x^2$ − 5 $x^2$ =

39.

e. 4 $x^y$ 3 ÷ ( 4 $x^2$ × 2 ) =

2.

f. ( 9 $x^2$ − 3 × 6 ) ÷ ( 1 2 + ( 4 − 1 ) ) =

9.

g. ( ( 1 2 − 6 ) × ( 1 2 − 6 ) ) ÷ ( 3 + 3 ) $x^y$ 2 =

3.

h. ( 3 $x^2$ × 4 + 3 × 2 ) ÷ ( 1 6 ÷ 4 + 8 + 2 ) =

3.

## Now Try This

4. Answers may vary. A sample answer is given.

MATH → MATS → MAPS → MOPS → TOPS

## Looking Back

5. Brackets
   Exponents
   Division
   Multiplication
   Addition
   Subtraction

## Section 3: Activity 5

1. a. $24\ 000\ 000 = 2.4 \times 10^{7}$ b. $0.000\ 000\ 43 = 4.3 \times 10^{-7}$

   c. $54\ 600\ 000\ 000 = \mathbf{5.46} \times 10^{10}$ d. $0.000\ 000\ 000\ 039 = \mathbf{3.9} \times 10^{-11}$

   e. $147\ 000\ 000 = \mathbf{1.47} \times 10^{8}$ f. $0.000\ 000\ 083 = \mathbf{8.3} \times 10^{-8}$

2. a. The number is not multiplied by a power of 10. It should be $4.8 \times 10^{2}$.

   b. The number is not between 1 and 10. It should be $5.2 \times 10^{-8}$.

   c. The number is written in scientific notation.

   d. The number is written in scientific notation.

   e. The number is not between 1 and 10. It should be $4.8 \times 10^{14}$.

3. a. $4.8 \times 10^{10}$ b. $7.2 \times 10^{-9}$ c. $1 \times 10^{15}$

4. a. 712 000 000 b. 0.000 000 000 42

   c. 100 000

5. $13\ 000\ 000 = 1.3 \times 10^{7}$

   The temperature at the centre of the Sun is about $1.3 \times 10^{7}\,°C$.

6. $0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 001\ 67 = 1.67 \times 10^{-24}$

   The mass of a hydrogen atom is about $1.67 \times 10^{-24}$ g.

7. 1 8 7 0 0 0 × 5 6 7 0 0 0 =

   1.06029 11

8. • 0 0 0 6 ÷ 3 0 0 0 0 =

   2. -08

   **Note:** Some calculators may not present the answer in scientific notation if all the digits fit in the display. In this case, the answer displayed will be 0.000 000 02.

9. a. $4.35 \times 10^{6}$

   b. $3.27 \times 10^{-5}$

10. The answer to question 7 is displayed in scientific notation because it has too many digits to fit on the calculator display.

11. Most scientific calculators allow a maximum of ten digits to be entered. Therefore, the largest number that can be displayed on most scientific calculators is as follows.

    `9999999999`

12. a. `0.03`

    b. `3.-03`

    Some calculators may not display this answer in scientific notation. Some may display the following.

    `0.003`

    c. `3.-04`

    Some calculators may not display this answer in scientific notation. Some may display the following.

    `0.0003`

    d. `0.03`

    e. `3.-03`

    Some calculators may not display this answer in scientific notation. Some may display the following.

    `0.003`

    f. `3.-04`

    Some calculators may not display this answer in scientific notation. Some may display the following.

    `0.0003`

13. If the result contains two or more zeros before a non-zero digit, then it is displayed in scientific notation. (This is true for most scientific calculators.)

14. a. $(3.91 \times 10^{11}) \times (1.84 \times 10^{-19}) = (3.91 \times 1.84) \times (10^{11} \times 10^{-19})$

$$= 7.1944 \times 10^{-8}$$

If you used the scientific calculator, your keystrokes would be

3 • 9 1 EXP 1 1
× 1 • 8 4 EXP 1 9 +/− =

7.1944-08

b. $(8.14 \times 10^{-4}) \times (9.25 \times 10^{6}) = (8.14 \times 9.25) \times (10^{-4} \times 10^{6})$

$$= 75.295 \times 10^{2}$$
$$= 7.5295 \times 10^{3}$$

8 • 1 4 EXP 4 +/−
× 9 • 2 5 EXP 6 =

7529.5

$7529.5 = 7.5295 \times 10^{3}$

c. $(7.73 \times 10^{-5}) \times (2.68 \times 10^{-2}) = (7.73 \times 2.68) \times (10^{-5} \times 10^{-2})$

$$= 20.7164 \times 10^{-7}$$
$$= 2.071\,64 \times 10^{-6}$$

7 • 7 3 EXP 5 +/−
× 2 • 6 8 EXP 2 +/− =

2.07164-06

15. $(1.4 \times 10^{9}) \div (0.25) = (1.4 \div 0.25) \times 10^{9}$

$$= 5.6 \times 10^{9}$$

There is about $5.6 \times 10^{9}$ kg of gold in the oceans.

16. $(1.429 \times 10^{9}) \div (1.08 \times 10^{9}) = (1.429 \div 1.08) \times (10^{9} \div 10^{9})$

$$= 1.323\,\overline{148} \times 10^{0}$$
$$\doteq 1.323$$

It takes light about 1.323 h to travel from the Sun to Saturn.

## Looking Back

17. Scientific notation allows you to write very large or very small numbers concisely. It makes them much easier to read, rather than having to count a large number of zeros.

# Credits

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16 Nova Development Corporation
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101 PhotoDisc, Inc.

# Puzzle

Connect the points in order, as you move down the columns of coordinates. Do not connect points separated by "Lift Pencil." Shade in areas formed by points inside a box.

(−2, −10)
(−1, −9)
(−7, −7)
(−9, −5)
(−10, −5)
(−9, −4)
(−10, −4)
(−8, −2)
(−10, −2)
(−7, 0)
(−5, 4)
(−6, 6)
(−10, 10)
(−11, 16)
(−11, 18)
(−9, 18)
(−5, 13)
(−3, 6)

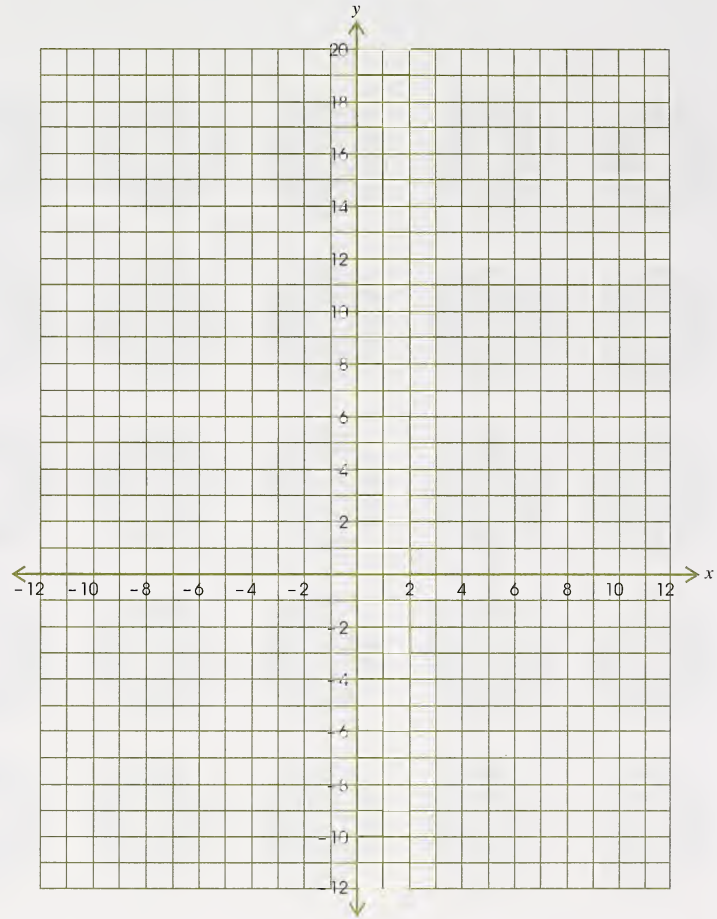
(−1, 6)
(2, 13)
(7, 16)
(10, 17)
(10, 16)
(8, 12)
(1, 5)
(2, 4)
(4, 0)
(6, 0)
(8, −1)
(7, −1)
(8, −3)
(7, −2)
(8, −4)
(7, −4)
(5, −7)
(2, −9)

(3, −10)
Lift Pencil
(−5, 7)
(−9, 11)
(−10, 17)
(−6, 12)
(−5, 7)
Lift Pencil
(1, 7)
(2, 11)
(4, 13)
(9, 16)
(6, 12)
(1, 7)
Lift Pencil
(0, 2)
(−1, 4)
(−3, 2)
(−2, 0)

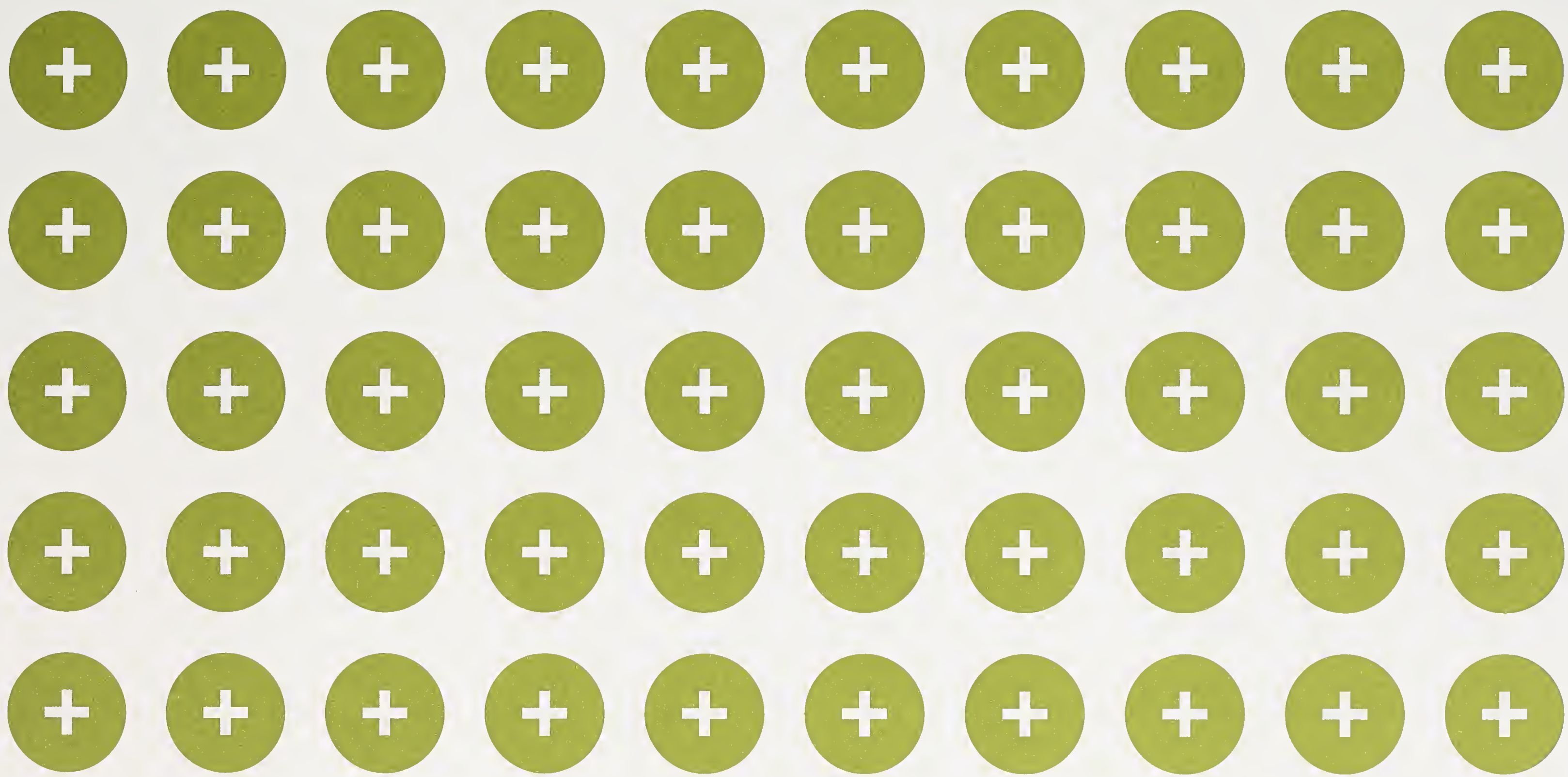
(−1, 0)
(0, 2)
(−1, 2)
(−2, 1)
(−1, 0)
Lift Pencil
(3, 2)
(1, 4)
(1, 0)
(3, 0)
(3, 1)
(2, 0)
(2, 2)
(3, 2)
(3, 1)
Lift Pencil
(−7, 0)
(1, −1)

(3, −1)
(2, −2)
(1, −1)
Lift Pencil
(0, −3)
(4, −3)
(3, −6)
(1, −6)
(0, −5)
(−2, −5)
(−3, −4)
(−1, −3)
(0, −3)
(1, −6)
Lift Pencil
(2, −6)
(1, −7)
(0, −7)
(−2, −5)

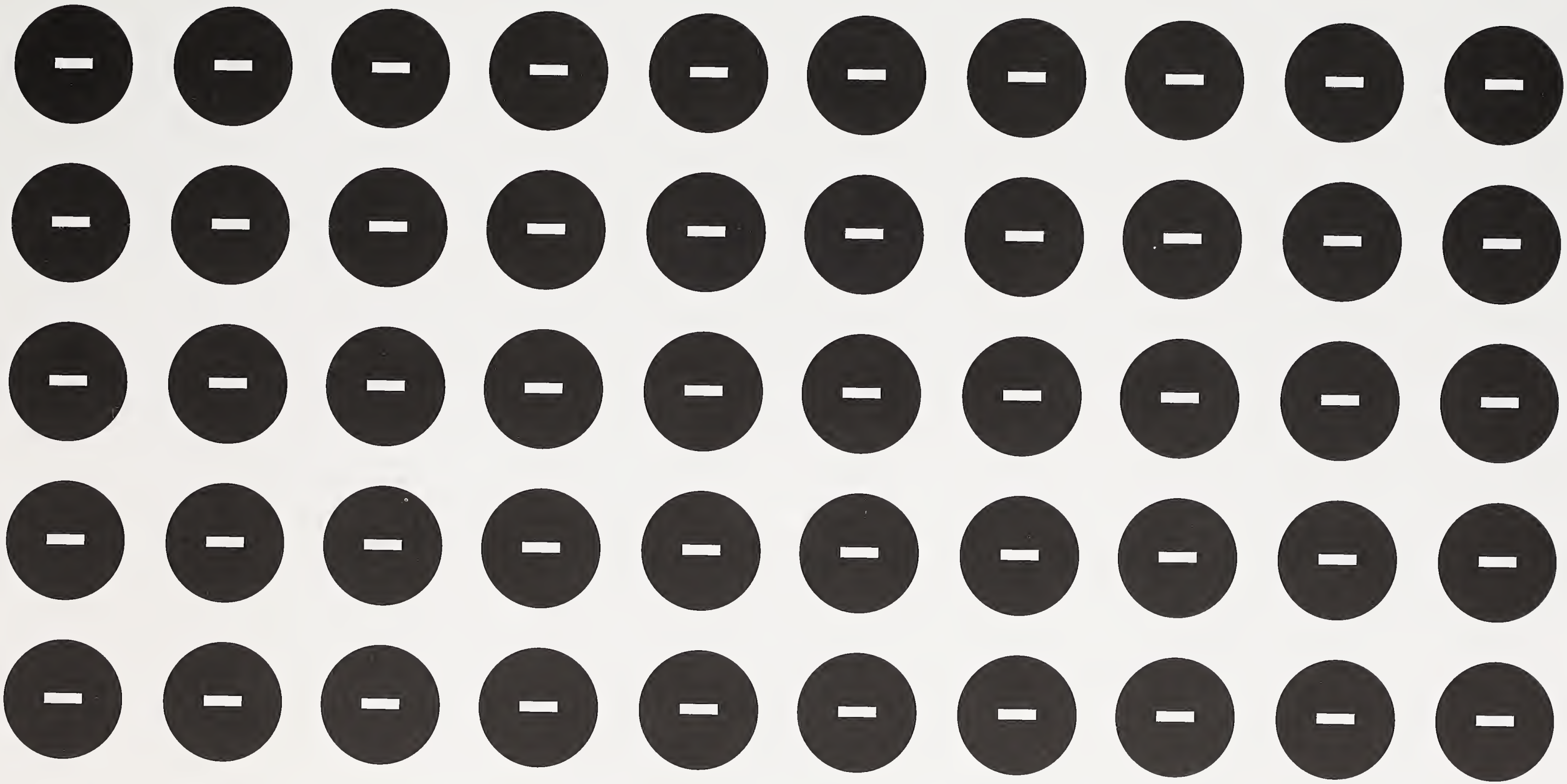
Lift Pencil
(2, −3)
(2, −5)
Lift Pencil
(−1, −5)
(0, −6)
Lift Pencil
(4, −2)
(11, 1)
Lift Pencil
(4, −2)
(11, −2)
Lift Pencil
(0, −2)
(−7, −1)
Lift Pencil
(0, −2)
(−8, −4)



Positive Counters



## Negative Counters



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